

Understanding Private Fund Performance

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July 2024

KEY TAKEAWAYS

- ▶ This paper studies the net performance of 6,000 private funds from 1980 to 2022, covering buyout, venture, credit, and real estate funds.
- ▶ There is wide dispersion in funds' lifetime performance, while performance relative to public markets is highly dependent on the choice of benchmark.
- ▶ Private funds in aggregate have provided diversification benefits to public investors.

We study the net performance of 6,000 private funds from 1980 to 2022, covering buyout, venture, credit, and real estate funds. We find wide dispersion in funds' lifetime performance in all asset classes. Performance relative to public benchmarks depends crucially on the choice of benchmark, and the average fund's public market equivalent is between 0.81x and 1.13x relative to style indices. Periodic returns are markedly more correlated with public factors following the adoption of fair value accounting, though a considerable fraction of their variation remains unexplained. Since private funds expand public investors' opportunity set, this suggests considerable diversification benefits.

^{*} Corresponding author. We thank Wei Dai, Bob Merton, Robert Novy-Marx, Ludovic Phalippou, Savina Rizova, Morten Sørensen and participants at the 7th Private Markets Research Conference in Lausanne for helpful comments and suggestions. Kaitlin Hendrix (kaitlin.hendrix@Dimensional.com) and Mamdouh Medhat (mamdouh.medhat@dimensional.com) are at Dimensional Fund Advisors.

Introduction

Private funds currently account for about 9% of the global investable universe in equities, fixed income, and real estate.¹ Investors are increasingly looking to private funds for alternative sources of returns and diversification relative to public markets.

Understanding the risk and return profiles of private funds is crucial for investors, wealth managers, and consultants. Yet, simply measuring the performance of private funds requires specialized methods due to the illiquid nature of their holdings. The same is true for benchmarking their performance to public investments. There is an active academic literature, both theoretical and empirical, on the performance of private funds.

This paper sheds new light on private fund performance using a sample of more than 6,000 North American funds from 1980 to 2022 that covers the four major asset classes: buyout, venture capital, private credit, and private real estate. We study absolute performance, performance relative to public benchmarks, and correlations between private and public investments. Our data are from the MSCI Private Capital Universe (formerly known as the Burgiss Manager Universe), which is sourced directly from outside investors (i.e., limited partners) and considered more comprehensive and of higher quality than other data sets.² We have access to averages and percentiles of quarterly cash flows and valuations at the vintage level (i.e., for groups of funds raised in a calendar year) but not for individual funds or managers. All cash flows and valuations are net of all fees, allowing us to take the view of outside investors. Our main contribution relative to the extant literature is to provide fresh evidence on private fund performance using comprehensive, high-quality data covering multiple asset classes.

We find substantial dispersion in lifetime absolute performance across funds in all asset classes. As is standard in the literature and among practitioners, we measure absolute performance using the total value to paid-in capital (TVPI, defined as distributions plus net asset value relative to contributions) and the internal rate of return (IRR, the breakeven discount rate on net cash flows and net asset value). Both measures indicate large differences between funds in the top and bottom of the performance distribution. **Figure 1** illustrates the dispersion using weighted average lifetime TVPI. Depending on the asset class, average lifetime TVPI is between 1.34x and 1.81x, but all asset classes have a wide range of outcomes: The 95th percentile is between 2.08x and 4.24x, while the 5th percentile is between 0.81x and 0.36x. We argue this dispersion exacerbates the manager selection problem because "holding the market" can be difficult in private allocations.

Table A1 in the Appendix shows a detailed breakdown of the size of the global investable universe by asset class as of December 31, 2022, based on data from MSCI, Bloomberg, FTSE, and Preqin.

See, e.g., Harris, Jenkinson, and Kaplan (2014, 2016); Brown, Harris, Jenkinson, Kaplan, and Robinson (2015); Kaplan and Sensoy (2015); Harris, Jenkinson, Kaplan, and Stucke (2023); Korteweg and Nagel (2023).

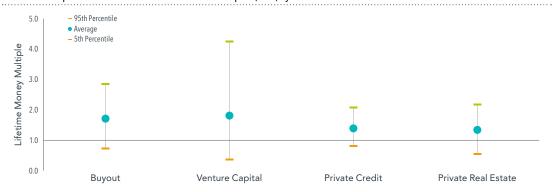


FIGURE 1: Dispersion in Lifetime Total Value to Paid-In Capital (TVPI) by Asset Class

Past performance is no guarantee of future results.

In USD. This figure shows weighted averages of statistics from the sample distribution of lifetime total value to paid-in capital (TVPI). The averaging is across vintages (i.e., groups of funds raised in the same calendar year) with weights determined by inflation-adjusted total committed capital (in 202204 dollars). The statistics are the simple average and the 5th and 95th percentiles. TVPI is total distributions plus net asset value relative to total contributions. All cash flows and net asset values are net of management fees and carried interest. See Appendix L for disclosures. The sample is all North American closed-end private funds reporting in USD from the MSCI Private Capital Universe, excluding funds of funds. Data are quarterly, end with 2022, and start in 1980 for venture capital, 1986 for buyout, and 1993 for private credit and private real estate.

We also find that whether private funds deliver a premium relative to public investments depends crucially on the choice of benchmark. We measure relative performance using Kaplan and Schoar's (2005) "public market equivalent" (KS-PME, a benchmark-adjusted TVPI with strong theoretical support) and Gredil, Griffiths, and Stucke's (2023) "direct alpha" (DA, a benchmark-adjusted IRR with the same theoretical foundation as KS-PME). Both suggest the choice of benchmark is highly influential. As an example, consider buyouts, which Figure 1 shows have an average lifetime TVPI of 1.71x. When benchmarked against the S&P 500 Index, the corresponding average lifetime KS-PME is 1.19x, i.e., above 1, indicating outperformance relative to the index. However, when benchmarked against small cap value stocks, KS-PME drops to 0.96x, indicating underperformance. To the extent private funds deviate from broad market exposure, style benchmarks may be more relevant representations of opportunity cost (Phalippou 2014; Sørensen and Jagannathan 2015). We find similar results across asset classes: Relative to style benchmarks, average lifetime KS-PME is 0.81x–1.13x.

Lastly, by holding unlisted assets, private funds expand public investors' opportunity set. They may therefore offer genuine diversification benefits, provided their returns are not perfectly correlated with listed asset returns. To test this, we use factor regressions. Specifically, we determine the extent to which public market factors explain aggregate periodic private fund returns, similar to the hedge fund literature.³ Because periodic returns partly reflect managers' valuations of ongoing investments, they tend to exhibit nonsynchronicity ("lagging") and lower volatility ("smoothing") relative to listed asset returns, which can understate factor exposures. Adjusting for these biases, we find that the explanatory power rises sharply for all asset classes after the adoption of fair value accounting in 2007. Nonetheless, a considerable fraction of the variation remains unexplained over this latter period, and the results are robust to excluding the 2008–2010 crisis years. As such, our analysis suggests private funds have in aggregate offered meaningful diversification benefits to public investors, even in more recent decades.

^{3.} See, e.g., Asness, Krail, Liew (2001); Fung and Hsieh (2002, 2004); Argawal and Naik (2004); Germantsky, Lo, and Makarov (2004); Bali, Brown, and Caglayan (2012, 2014); Bollen, Joenvaara, and Kauppila (2021).

The remainder of the paper is organized as follows. Section 2 provides a primer on private funds, describes the data, and illustrates the methodology. Section 3 documents the dispersion in absolute performance and the sensitivity of relative performance to the choice of benchmark. Section 4 shows factor regressions and discusses diversification benefits. Section 5 concludes.

2. Data, Methodology, and Single-Vintage Example

In this section, we describe our data and illustrate our methodology using a single-vintage example. We start, however, with a primer on the structure and terminology of private funds. Private investing aficionados may wish to skip to Section 2.2.

2.1 Private fund structure and terminology

Private funds are typically incorporated as limited partnerships between a fund management company, referred to as the *general partner* (GP), and outside investors, each referred to as a *limited partner* (LP). Historically, LPs have been institutional investors such as sovereign wealth funds, pension plans, and endowments. Partnerships usually last seven to 10 years after fundraising, though longer agreements and extensions are not uncommon.

During fundraising, GPs seek capital commitments from LPs, typically subject to a minimum commitment. GPs often coinvest with LPs (say, 2% of the total commitment). Funds that end their fundraising within a given calendar year belong to that year's *vintage*. GPs can call upon the committed capital at their discretion, though most calls are within the first three to four years. Any committed but uncalled capital is called *dry powder*. Dry powder can be a performance drag for LPs as it typically has to sit in near-cash assets.

GPs use the paid-in capital along with any raised debt (i.e., leverage) to enter investment positions, or deals, as owners, shareholders, or creditors in unlisted companies or projects. A GP's mandate is to create value for LPs through various activities, including engagement with company or project management, financial or operational engineering, and asset restructuring. LPs receive distributions, also at the GP's discretion, especially as the GP exits deals. GPs charge a management fee (say, an annual 2% of committed or managed capital) and a performance fee, called *carried interest* (say, 20% of profits).

All cash outflows from the fund to stakeholders are subject to a tiered *seniority* or *waterfall* structure. First, distributions go toward paying the GP's management fee and servicing any debt. In fact, additional capital may be called from LPs for these purposes. Second, distributions go toward returning the called capital back to the partners, which may include the GP. Third, LPs receive all profits until they achieve a certain *preferred return* or *hurdle rate* (say, an 8% internal rate of return). Fourth, after LPs achieve their hurdle rate, the GP enters a *catch-up* tier, where she receives a majority share of profits (often 80-100%) until she achieves a predetermined

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share of total profits (carried interest). Lastly, after the GP has caught up, LPs and the GP enter a *profit-sharing* tier, where GPs receive a share of profits equal to the rate of carried interest. The waterfall structure can be *European* (also known as global), where the LPs' hurdle rate is at the fund level, or *American* (also known as *deal-by-deal*), where the hurdle rate is at the deal level. The latter is more favorable to the GP as it distributes carried interest faster. Since the fund's ultimate return is known only once all deals are exited, any distributions to the GP above her contributed capital are "carried" throughout the fund's lifetime and subject to *clawbacks*.

Because unlisted assets lack readily available clearing prices, the value of a private investment must be estimated prior to an exit. When communicating periodic performance to LPs, GPs report valuations of ongoing investments, typically based on historical costs, accounting appraisals, and comparable deals or listed assets. LPs commonly receive quarterly financial statements that include the fund's *net asset value* (NAV). The NAV reflects a *remaining* or *residual* value, computed as the valuation of ongoing investments less management fees and the values of any debt and carried interest.

Compared to listed asset returns, periodic private fund returns exhibit seemingly lower volatility, or *smoothing*, and delayed reactions, or *lagging*. This is a direct consequence of the appraisal-based valuations of ongoing investments, as reflected in a private fund's NAV. The gap was somewhat bridged by the adoption of US and international accounting standards on *fair value measurement* in 2006–2008, which include *mark-to-market* provisions.⁴ Still, since some lagging and smoothing is unavoidable for unlisted assets, working with periodic private fund returns may require adjustments or special methods.

An LP's shares inherit the illiquidity of a private fund's investments. Private funds tend to have multiyear *lockup periods*, during which shares cannot be redeemed, followed by periodic redemption limits. Further, while the NAV in principle reflects the potential resale value of LP shares, early exits can entail a discount, or *haircut*, to the NAV, whether in the form of redemptions or sales in the secondary market for private fund shares.⁵

The largest and most prominent private asset classes are buyout (BO), venture capital (VC), private credit, and private real estate. BOs typically take levered equity stakes in relatively mature, or *late stage*, private companies, or controlling stakes in listed companies with the intent of taking them private. VCs take equity stakes in startups and relatively young, or *early stage*, private companies. Private credit funds act as direct (nonbank) lenders to companies, projects, and other private funds (primarily BOs). Lastly, private real estate funds typically acquire, own, and manage commercial or residential real estate properties.

^{4.} The US accounting standard on fair value measurement is FASB GAAP ASC 820, formerly SFAS 157, effective November 15, 2007. The corresponding international standards are the amended IAS 39, effective November 15, 2005 (now replaced by IFRS 9), and IFRS 13, effective January 1, 2013. The International Private Equity and Venture Capital Valuation (IPEV) guidelines were updated in November 2006 to be consistent with both IAS 39 and SFAS 157. The most recent update was in December 2022 (see privateequityvaluation.com).

^{5.} Boyer, Nadauld, Vorkink, and Weisbach (2023) use data from a large intermediary in the secondary private equity market from 2006 to 2018 and find that "funds on average transact at a discount relative to NAV" with "the overall average [...] corresponding roughly to a 17% discount [...] and the median [...] corresponding to a 10% discount" (p. 854). Sørensen, Wang, and Yang (2014) solve and calibrate a structural model of a private investment to determine the conditions under which performance is sufficient to compensate LPs for risk, illiquidity, and the total fees charged by GPs.

2.2 MSCI Private Capital Universe

The MSCI Private Capital Universe (PCU, formerly known as the Burgiss Manager Universe) provides detailed histories of private fund cash flows and valuations. The data are sourced directly from LPs who use the associated platform for fund accounting and performance monitoring. As of December 2023, more than 1,000 LPs use the platform.

The PCU includes only closed-end funds with GP discretion over cash flows and excludes open-end funds as well as direct investments and co-investments. The data are typically updated in a timely manner due to most LPs' need for quarterly reporting. MSCI cross-checks different LPs' inputs for the same fund and rescales the data to be representative of the full fund. All cash flows and valuations are net of management fees and carried interest. Importantly, no data are sourced through surveys, voluntary GP submissions, web scraping, or the Freedom of Information Act. This makes the PCU less prone to the sample-selection and self-reporting biases identified in other databases (e.g., Stucke 2011; Harris, Jenkinson, and Kaplan 2014, 2016). As such, it has become popular in academic studies (recent examples include Harris, Jenkinson, Kaplan, and Stucke 2023; Gredil, Griffiths, and Stucke 2023; and Korteweg and Nagel 2023).

The platform provides access to a limited set of statistics based on the fund-level data, but not to the underlying fund-level data. According to MSCI, this is to ensure anonymity of the funds, their GPs, and the LPs who provide the data. Users can filter the full fund universe by asset class, vintage year, fund domicile, etc., but the platform will not display any output if the filtering results in a subset with fewer than five funds. Given a filtering (say, the 2007 vintage of North American BOs) and a variable of interest (say, paid-in capital), the output is a quarterly time series of certain statistics from the variable's sample distribution across funds: the simple average as well as the 5th, 25th, 50th, 75th, and 95th percentiles. We also see total committed capital and total number of funds by vintage.

The restrictions on what is available to us through the PCU means we are limited in what we can study with the data. First, we are unable to conduct any fund- or GP-level analyses. This precludes us from studying, for instance, the sources of GP-added value and whether there is persistence in GP performance. Second, we have to focus on the broad asset classes because a too granular filtering (e.g., splitting private credit into senior, mezzanine, and distressed debt) often results in fewer than five funds per vintage, especially in the early vintages, and hence no output. Lastly, any variable we compute ourselves, such as a relative performance measure, must be based on the average values of cash flows and NAVs. This is because the 5th percentile of cash flows, for instance, cannot simply be combined with the 5th percentile of NAVs in a computation. In particular, we can study average relative performance, but not the dispersion in relative performance.

^{6.} In exceptional cases, MSCI allows academics limited access to fund- and GP-level data subject to nondisclosure agreements. MSCI requires the analysis of such data to be conducted by the academics using its computers at its premises or by its employees using code provided by the academics. Recent examples are Brown, Ghysels, and Gredil (2023), who "nowcast" fund-level weekly NAVs and returns, and Harris, Jenkinson, Kaplan, and Stucke (2023), who study performance persistence. The latter find that, conditional on the interim performance of an existing fund at the time a new fund is raised, there is "little evidence of persistence for buyouts, especially post-2000" but some "persistence for VC funds, though it declines post-2000" (abstract).

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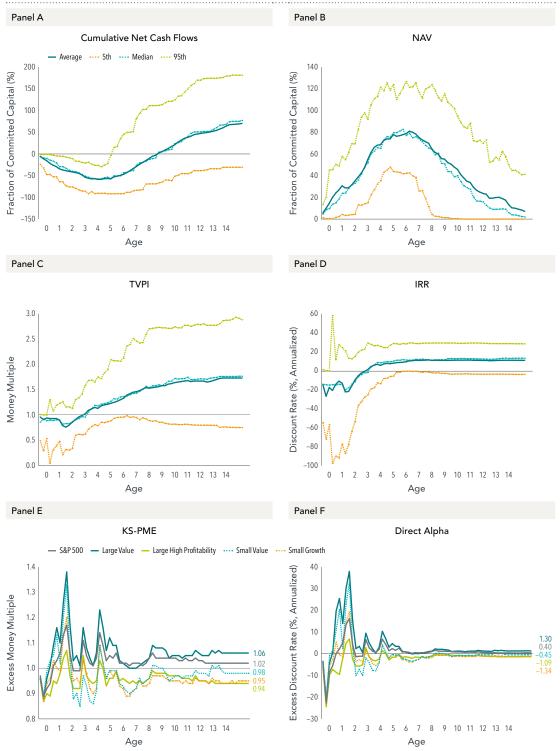
Our sample consists of closed-end private funds domiciled in North America with cash flows and NAVs in US dollars (USD). The total number of funds across all vintages is 6,021: 1,575 BOs, 2,509 VCs, 913 credit funds, and 1,024 real estate funds. The first vintage year is 1980 for VCs, 1986 for BOs, and 1993 for both credit and real estate. Figure B1 in the Appendix shows inflation-adjusted total committed capital (in 2022Q4 dollars) and the number of funds by vintage for each asset class. Our sample period ends with 2022Q4.

We adopt two conventions when measuring vintage age. First, the data for each vintage start from the end of Q1 of the vintage year, but committed capital and the number of funds are not fixed until the end of Q4 of the vintage year. Hence, we measure vintage age relative to the end of Q4 of the vintage year, when all funds are fully raised. This means each vintage's first quarterly observation is at age -0.75 years. Second, because the PCU does not report when a vintage has been fully liquidated, we define this as the first quarter after which we see no change in the sample distribution of NAVs. By this time, the average NAV is typically at or very close to zero. In our sample, this occurs at a maximum age of 15 years for BOs, 17 years for VCs, 12 years for credit, and 14 years for real estate.

In the following, we illustrate the data structure and our methodology using the 2007 BO vintage. We choose this vintage because it is relatively recent and the underlying funds appear fully liquidated by 2022Q4 (the end of our sample period). The PCU reports \$172 billion in committed capital (in 2022Q4 dollars) and 68 funds for this vintage (Figure B1). Figure 2 illustrates the data we observe directly from the PCU (Panels A–D) and the relative performance measures we compute ourselves (Panels E–F). Table 1 shows formulas for the absolute and relative performance measures we use. Note that these measures ignore the potential performance drag for LPs from dry powder sitting in near-cash assets.

^{7.} Note that we do not have data for the 1991 BO vintage and the 1995 private credit vintage because the PCU reports only four and three funds in these vintages. Note also that the PCU's coverage of vintages starts earlier for VCs than for BOs. Gompers and Kaplan (2022, p. 8-9) review the history of the private equity industry and note that the first BOs grew out of the already existing VC industry in the late 1970s, helped in part by the creation of the public high-yield bond market.

FIGURE 2: Single-Vintage Example-2007 BOs



Past performance is not a guarantee of future results.

In USD. This figure illustrates the data we observe directly from the PCU (Panels A—D) and the relative performance measures we compute ourselves (Panels E-F) for the 2007 vintage of BOs. The PCU reports \$172 billion in committed capital (in 202204 dollars) and 68 funds for this vintage. Each panel shows statistics from the sample distributions of selected variables as a function of vintage age is measured in years relative to 200704 (when all funds in the vintage are fully raised). TVPI, IRR, KS-PME, and DA are lifetime values. All cash flows and NAVs are net of management fees and carried interest. IRRs and DAs over periods shorter than one year are not annualized. See **Appendix K** for index definitions and **Appendix L** for disclosures. Data are quarterly and cover 200701 through 202204.

2.3 Cash flow and valuation data

Panel A of Figure 2 plots cumulative net cash flows (distributions minus paid-in capital, expressed as a percentage of committed capital) as a function of vintage age (years relative to 2007Q4). Cumulative net cash flows exhibit a characteristic "J" shape: negative early on as capital is called by GPs; then gradually moving into positive territory as GPs distribute capital back to LPs. LPs invested in the average fund had to wait eight years to break even in terms of net cash flows. Panel B shows that NAVs exhibit a similarly distinctive "hump" shape: sharply increasing early on as GPs call capital and enter deals, then plateauing before gradually decreasing as GPs exit deals and return profits to LPs.

2.4 Absolute performance measures

The first column of **Table 1** shows the absolute performance measures, which we get directly from the PCU. The total value to paid-in capital (TVPI) is a money multiple that expresses cumulative distributions plus NAV as a fraction of cumulative paid-in capital. A value above 1 indicates that, adjusted for any remaining value, the fund distributed more capital than it called, and vice versa for a value below 1.

Panel C of Figure 2 shows lifetime TVPI as a function of age for the 2007 BO vintage. For the average fund, performance is negative early on, when distributions and valuations cannot keep up with capital calls, but turns positive after about three years and ultimately converges to a lifetime (15-year) multiple of 1.73. That is, the average fund made LPs whole plus an additional 73 cents per \$1 of called capital. There is visible dispersion across funds: For this vintage, the 5th and 95the percentiles of lifetime TVPI are 0.75 and 2.93.

The internal rate of return (IRR) is the annualized breakeven discount rate on net cash flows and NAV; that is, the discount rate that implies a zero present value of net cash flows and NAV.⁸ There are several caveats with interpreting IRR as a "return on capital" (e.g., Phalippou 2021, chap. 15). For one, it assumes distributions are reinvested at a rate equal to the IRR, whether positive or negative, which may not be realistic. Furthermore, it can be inflated by large, early distributions, possibly through the use of leverage, because later cash flows are discounted more heavily. TVPI does not assume a reinvestment rate and is independent of the timing of cash flows, but ignores the time value of money and is not expressed as a rate.

Panel D of **Figure 2** shows that the IRR behaves similarly to the TVPI but converges more quickly to its ultimate value. For this vintage, the average fund's annualized lifetime IRR is 10.88%, while the 5th and 95th percentiles are -3.96% and 28.25%. Note that there is no straightforward relation between TVPI and IRR. Average lifetime TVPI suggests an annualized return of $1.73^{1/15} - 1 = 3.72\%$, less than the 10.88% IRR because TVPI ignores the timing of cash flows, while IRR is more reflective of early cash flows. Kocis, Bachman, Long, and Nickels (2009)

^{8.} To see the intuition behind the IRR, consider its definition in **Table 1** for a one-period fund, i.e., for t=1. We get $0 = NCF_0 + \frac{NCF_1 + NAV_1}{1 + IRR_{0,1}}$, where $NCF_s = DST_s - PIC_s$ is net cash flow (distribution minus paid-in capital) at time s=0,1 with an initial investment of $NCF_0 = -NAV_0$. This implies $IRR_{0,1} = \frac{NCF_1 + NAV_1}{NAV_0} - 1$, which is the rate of return on an investment valued at the NAV and paying a dividend equal to the net cash flow.

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TABLE 1: Absolute and Relative Performance Measures

This table shows formulas for computing absolute and relative performance measures. DST_s is capital distribution at time s, PIC_s is paid-in capital, NAV_s is net asset value, $NAV_s = DST_s - PIC_s$ is net cash flow, and B_s is the benchmark level.

	Absolute	Relative			
	Total Value to Paid-In Capital	Kaplan-Schoar Public Market Equivalent			
Money Multiple	$\text{TVPI}_{0,t} = \frac{\sum_{s=0}^{t} \text{DST}_{s} + \text{NAV}_{t}}{\sum_{s=0}^{t} \text{PIC}_{s}}$	$\text{KS PME}_{0,t} = \frac{\sum_{s=0}^{t} \text{DST}_{s} \frac{B_{t}}{B_{s}} + \text{NAV}_{t}}{\sum_{s=0}^{t} \text{PIC}_{s} \frac{B_{t}}{B_{s}}}$			
	Distributions plus remaining value elative to contributions (also known as multiple on invested capital, or MOIC)	Benchmark-adjusted distributions plus remaining value relative to benchmark-adjusted contributions			
	Internal Rate of Return	Gredil-Griffiths-Stucke Direct Alpha			
Discount Rate	$0 = \sum_{s=0}^{t} \frac{NCF_s}{(1 + IRR_{0,t})^s} + \frac{NAV_t}{(1 + IRR_{0,t})^t}$	$0 = \sum_{s=0}^{t} \frac{\text{NCF}_{s} \frac{B_{t}}{B_{s}}}{(1 + \text{DA}_{0,t})^{s}} + \frac{\text{NAV}_{t}}{(1 + \text{DA}_{0,t})^{t}}$			
	Annualized breakeven discount rate on net cash flows and remaining value	Annualized breakeven discount rate on benchmark-adjusted net cash flows and remaining value			

combine TVPI and IRR to estimate a fund's *duration*, or effective holding period.⁹ Since $1.73^{1/5.31} - 1 \approx 10.88\%$, the duration is roughly 5.31 years.

2.5 Kaplan-Schoar public market equivalent

The second column of Table 1 shows the relative performance measures we use. Kaplan and Schoar's (2005) *public market equivalent* (KS-PME) is a benchmark-adjusted TVPI, where distributions and paid-in capital are multiplied by the growth of \$1 in the benchmark from the time of each cash flow to the time KS-PME is computed. Intuitively, a multiple above 1 means the fund outperformed an investment in the benchmark matched on cash-flow timing and amount. More precisely, KS-PME has two equivalent interpretations:

- Suppose the LP finances capital calls by selling positions in the benchmark and reinvests distributions back into the benchmark. Then KS-PME is the value of reinvested distributions plus NAV relative to the opportunity cost of the sold positions. A ratio above 1 means the fund generated distributions at a higher rate than the cost of financing capital calls. This interpretation is due to Kaplan and Schoar (2005).
- Suppose the LP determines the present value (PV) of a given cash-flow stream by discounting at the benchmark's cumulative return, for instance, because the benchmark captures the LP's views on risk, expected return, and opportunity cost. Then KS-PME estimates the PV of distributions and NAV relative to the PV of paid-in capital (to see this, multiply both the numerator and denominator by B_0/B_t and write $B_0/B_s = 1/(1 + r_{0,s}^B)$, where $r_{0,s}^B = B_s/B_0 1$). A ratio above 1 means the fund has a positive PV. This interpretation is due to Sørensen and Jagannathan (2015).

^{9.} Motivated by the relation between present and future values, Kocis, Bachman, Long, and Nickels (2009) define duration at time t through $TVPI_{0,t} = (1 + IRR_{0,t})^{Dur_{0,t}}$, which means $Dur_{0,t} = log(TVPI_{0,t})/log(1 + IRR_{0,t})$. Phalippou and Gottschlag (2009) suggest a similar concept.

KS-PME avoids the issues with IRRs and has a strong link to valuation theory. Sørensen and Jagannathan (2015) show it corresponds to valuing cash flows using the stochastic discount factor (SDF) implied by Rubinstein's (1976) dynamic capital asset pricing model (CAPM), valid under a family of utility functions that nests logarithmic utility. While the standard CAPM implies a constant discount rate that depends on beta and the benchmark's expected return, the Rubinstein CAPM implies a *time-varying* discount rate equal to the benchmark's *realized* return. It implicitly accounts for systematic risk through the covariance between cash flows and benchmark returns and does not require estimating a beta, which is useful for illiquid assets. Sørensen and Jagannathan conclude that KS-PME "is valid regardless of the investment's beta, even if that beta changes over the life of the investment," and that it can be used to "evaluate risk-adjusted performance without explicitly calculating any betas or even knowing the risk of the underlying investments" (p. 44).

Some calculate a variant of KS-PME where the benchmark's realized return is scaled by a constant above 1. This is typically motivated as a "beta adjustment," despite the fact that the benchmark's realized return has no role in the discount rate implied by the standard CAPM. On this, Sørensen and Jagannathan (2015) write the following: "We are unaware of any formal asset-pricing model that justifies this risk adjustment, and it appears to 'double count' the systematic risk by simultaneously using the risk adjustments from both the standard CAPM and the Rubinstein CAPM" (p. 49). Empirically, the adjustments tend to have limited impact on KS-PME estimates. We therefore refrain from making arbitrary assumptions about betas or their time variation in our main analysis, and instead report such "adjusted KS-PMEs" as simple robustness checks in footnotes. As we discuss in Section 4, estimating private fund betas is nontrivial and an ongoing area of research.

2.6 The effects of leverage on KS-PME

Much of the discussion around betas stems from a concern that any outperformance of private funds hinges on their use of leverage. While leverage can increase KS-PME, it cannot in general inflate it from below 1 to above 1. Here, we demonstrate this using a numerical example, but **Appendix C.1** provides a formal derivation and additional details.

^{10.} Relative performance measures predating KS-PME include the Index Comparison Method (also called the Long-Nickels PME; Long and Nickels 1996) and the PME+ (Rouvinez 2003). PME+ uses a heuristic adjustment to resolve computational issues with the Long-Nickels PME. The same is true for the so-called modified PME (mPME; Cambridge Associates 2013). All three produce an IRR for a hypothetical investment in the benchmark to be compared with a fund's IRR. See Gredil, Griffiths, and Stucke (2023) for a discussion.

^{11.} Kortweg and Nagel (2016, 2021) show that the KS-PME is a special case of their so-called generalized public market equivalent (GPME) based on exponentially affine SDFs. Specifically, it is a GPME that avoids the estimation of SDF parameters by assuming Rubinstein (1976) preferences. Sørensen and Jagannathan (2015) argue that Rubinstein preferences "should be interpreted as general approximations of the economic environment and seem no more contrived than the assumptions required to derive the standard CAPM" (p. 49). They also stress, however, that KS-PME has "important limitations," including that "It is more useful as an ex post measure of past performance" and that it "provides the value at the margin, which is relevant for making a small additional investment [...] but may be insufficient for evaluating substantial asset allocation decisions" (p. 48-49). See Kaplan and Sensoy (2015) and Gredil, Griffiths, and Stucke (2023) for more on the implications of Rubinstein preferences and the limitations of KS-PME.

^{12.} Robinson and Sensoy (2016) find that "using a [constant] β of 1.3 for buyout funds and 2.0 for venture capital funds" implies KS-PMEs that are "only slightly reduced relative to the standard [KS-PME]" (p. 11). Moreover, they plot KS-PMEs for this "beta" ranging from 0 to 3 and conclude that "reasonable [beta] choices have little effect on [KS-PME] estimates" (p. 11). Phalippou (2014) calibrates a beta of 1.3 for BO funds and finds that "the [KS-PME] of buyout funds is not very sensitive to beta" (p. 192).

Consider a one-period binomial tree with equally likely states. Suppose the benchmark return is either 40% or -20%, for an expected return of 10%. Under the Rubinstein CAPM, the PV of a \$1 in either state is $\frac{1}{2}(\frac{1}{1.4}+\frac{1}{0.8})=0.98$, so the risk-free rate is $\frac{1}{0.98}-1=1.81\%$. The GP can invest \$1,000 in a deal with a constant excess return, α , above the benchmark.

Without leverage, the LP contributes \$1,000. The PV of the unlevered deal is the PV of the distribution minus the PV of the contribution, $1000 \times \frac{1}{2} \left(\frac{1.4 + \alpha}{1.4} + \frac{0.8 + \alpha}{0.8} \right) - 1000$. Taking the ratio (instead of the difference) of these PVs gives the unlevered KS-PME, $\frac{1}{2} \left(\frac{1.4 + \alpha}{1.4} + \frac{0.8 + \alpha}{0.8} \right)$.

If the GP instead finances \$400 of the \$1,000 with debt, the LP's contribution is only \$600. Assume, for simplicity, the debt is risk-free (**Appendix C.2** considers risky debt). Using the risk-free rate and assuming the debt is fairly priced (i.e., creditors break even), the GP owes 400(1 + 1.81%) = 407.27, which has a PV of \$400. The PV of the levered deal is then

$$\frac{1}{2} \left(\frac{1000(1.4 + \alpha) - 407.27}{1.4} + \frac{1000(0.8 + \alpha) - 407.27}{0.8} \right) - 600$$

$$= \left[1000 \times \frac{1}{2} \left(\frac{1.4 + \alpha}{1.4} + \frac{0.8 + \alpha}{0.8} \right) - 400 \right] - [1000 - 400].$$

The two terms in square brackets are the PVs of the distribution and contribution adjusted for leverage. When α is independent of the financing mix and debt is fairly priced, the PV of the deal is the same with or without leverage. Levered KS-PME is then given by

$$\frac{1000 \times \frac{1}{2} \left(\frac{1.4 + \alpha}{1.4} + \frac{0.8 + \alpha}{0.8} \right) - 400}{1000 - 400}.$$

If $\alpha=-10\%$, unlevered KS-PME is 0.90 but levered KS-PME is an even lower 0.84 because the LP would be better off levering the benchmark. If $\alpha=0\%$, KS-PME is 1 with or without leverage. If $\alpha=10\%$, leverage *increases* KS-PME from 1.10 to 1.16 because the LP is better off in the levered deal. In general, levering with fairly priced debt pushes unlevered KS-PME further away from 1. Leverage cannot cause a crossing from below 1 to above 1 unless the GP (i) can use leverage to increase the deal's excess return or (ii) can obtain "cheap" debt. Sørensen, Wang, and Yang (2014) study the theoretical effects of cheap debt on private fund performance. In practice, LPs may not observe the debt or its pricing.

2.7 KS-PME for the 2007 BO vintage

Panel E of **Figure 2** shows KS-PMEs against several benchmarks for the average fund in the 2007 BO vintage.¹³ The first is the S&P 500 Index, which is a common BO benchmark among practitioners and in the literature. The corresponding lifetime KS-PME is 1.02, suggesting performance in line with the index.

^{13.} As discussed in Section 2.2, any variable we compute ourselves, such as KS-PME, must be based on the average values of cash flows and NAVs. Jensen's inequality implies that this is different from averaging fund-level KS-PMEs. A simple check, however, suggests that the Jensen effect is negligible in our sample. Specifically, the PCU does in fact report statistics from the sample distribution of KS-PME using the S&P 500 Index by vintage. For the 2007 BO vintage, the average reported by the PCU is 1.02, identical to ours. As we discuss in Section 3.2, this result generalizes when we average across BO and VC vintages in our sample.

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Phalippou (2014) suggests computing KS-PMEs for BO funds using style benchmarks because they may better capture investors' opportunity costs or risks not captured (unspanned) by a broad market index.¹⁴ We therefore compute KS-PMEs relative to four multifactor research indices that capture systematic differences in expected returns across stocks: a small cap value index, a small cap growth index, a large cap value index, and a large cap high-profitability index (see **Appendix K** for index definitions). For the average fund in the 2007 BO vintage, KS-PME suggests outperformance against large cap value (multiple of 1.06) but underperformance against small cap value, small cap growth, and large cap high-profitability (multiples of 0.98, 0.95, and 0.94, respectively).¹⁵

2.8 Gredil-Griffith-Stucke direct alpha

Gredil, Griffith, and Stucke's (2023) direct alpha (DA) retains the theoretical foundation of KS-PME but is an excess discount rate. Specifically, it is the IRR on benchmark-adjusted net cash flows and NAV using the same adjustment as KS-PME (see Table 1). To see the intuition behind DA, consider its definition for a one-period fund: $0 = \text{NCF}_0 \frac{B_1}{B_0} + \frac{\text{NCF}_1 + \text{NAV}_1}{1 + \text{DA}_{0,1}}$. Rearranging and using the expression for a one-period IRR (see Footnote 8), we have

$$DA_{0,1} = \frac{1 + IRR_{0,1}}{1 + r_0^2} - 1, \tag{1}$$

where $r_{0,1}^B = \frac{B_1}{B_0} - 1$ is the benchmark's one-period return. Though DA is useful, especially alongside KS-PME, it is ultimately an IRR and must be interpreted with caution. The benchmark adjustment means DA is less sensitive to the timing of distributions compared to IRR, but it still assumes a reinvestment rate equal to DA, whether positive or negative, which may not be realistic.

Panel F of Figure 2 shows DAs for the average fund in the 2007 BO vintage using the same benchmarks as for the corresponding KS-PMEs. Annualized lifetime DA relative to the S&P 500 Index is 0.40%. If we instead use the style benchmarks, it is 1.30% against large cap value, -0.45% against small cap value, -1.09% against small cap growth, and -1.34% against large cap high profitability. As such, DA reaffirms the conclusions based on KS-PME but provides an estimate of the annualized performance difference.

^{14.} Phalippou (2014) computes KS-PME using both research portfolios and live strategies as benchmarks. One advantage of using live strategies as benchmarks is that they are net of fees, whereas the indices and research portfolio we use as benchmarks are gross of fees.

^{15.} For the 2007 BO vintage, assuming a constant benchmark scaling factor of 1.3 (Phalippou 2014; Robinson and Sensoy 2016) implies generally lower KS-PMEs: 0.91 against the S&P 500, 0.97 against small cap value, 0.82 against large cap high-profitability, 0.89 against small cap value, and 0.84 against small cap growth.

3. Absolute and Relative Performance by Asset Class

This section shows our results for absolute and relative performance across all vintages.

3.1 Absolute performance and dispersion

Figure 3 plots, for each asset class, weighted averages of statistics from the sample distributions of lifetime TVPI and IRR as a function of vintage age. The averaging is across vintages with weights determined by inflation-adjusted total committed capital (in 2022Q4 dollars). We show the simple average, median, and 5th and 95th percentiles. Note that considering the extreme percentiles is relevant for understanding the full performance distribution, not least given the limited evidence of persistence documented by Harris, Jenkins, Kaplan and Stucke (2023), also using PCU data. Figure D1 in the Appendix shows the corresponding plots for cumulative net cash flows and NAVs.

In every asset class, the average fund had a TVPI above 1 and a positive IRR at the end of its lifetime. Still, the stark contrast between the top and bottom performers is evident using either measure. Lifetime TVPI, for instance, was on average between 1.34 (for private real estate) and 1.81 (for VC), but the 95th percentile was between 2.08 (for private credit) and 4.24 (for VC), while the 5th percentile was between 0.81 (for private credit) and 0.36 (for VC). The median is essentially indistinguishable from the average at all ages for both TVPI and IRR, except for VCs. Here, the averages are higher than the medians, with lifetime values of 1.81 vs. 1.24 for TVPI and 14.28% vs. 4.22% for IRR. In general, VC funds exhibit the most dispersion, while private credit funds exhibit the least. The statistics tend to converge to their long-term values at ages 3–4 years for IRR and 7–8 years for TVPI.

Figure E1 in the Appendix shows statistics for trailing 10-year TVPI and IRR by calendar year for each asset class. Since lifespans vary across asset classes and vintages, a 10-year horizon ensures a decent number of mature vintages per asset class while allowing for comparison across asset classes. The figure shows substantial dispersion within and across vintages in every asset class. The time-series variation is especially dramatic for VCs around the mid-2000s, where, for instance, TVPI went from 2 to 10 and back.

FIGURE 3: Absolute Performance Panel A: Buyout TVPI IRR Median 60 Discount Rate (%, Annualized) 40 2.5 20 Money Multiple 2.0 11.62 1.5 -20 -40 0.5 -60 0.0 10 11 12 13 14 Age Age Panel B: Venture Capital TVPI IRR 70 60 50 5.0 Discount Rate (%, Annualized) 4.0 40 30 20 Money Multiple 3.0 14.28 10 2.0 4.22 0 -10 1.0 -20 -30 -40 0.0 9 10 11 12 13 14 15 16 0 1 8 9 10 11 12 13 14 15 16 Age Panel C: Private Credit TVPI IRR 2.5 Discount Rate (%, Annualized) 20 2.0 Money Multiple 10 1.5 -10 -20 0.5 -30 -40 0.0 9 10 6 Age Panel D: Private Real Estate TVPI IRR 2.5 40 Discount Rate (%, Annualized) 20 2.0 Money Multiple 1.5 1.0 -20 0.5 -40 -60 10 11 12 13 10 11 12 13 Age Age

Past performance is not a guarantee of future results.

In USD. This figure shows weighted averages of selected statistics from the sample distributions of TVPI and IRR as a function of vintage age. The averaging is across vintages with weights determined by inflation-adjusted total committed capital (using US CPI and expressed in 2022Q4 dollars). Age is measured in years relative to the end of Q4 of the vintage year. All cash flows and NAVs are net of management fees and carried interest. IRRs over periods shorter than one year are not annualized. See Appendix L for disclosures. The sample is North American private funds covered by the PCU. Data are quarterly, end with 2022, and start in 1980Q1 for venture capital, 1986Q1 for buyouts, and 1993Q1 for private credit and private real estate.

The dispersion in performance exacerbates the problem of selecting a GP. Manager selection is in general complex, but there are special considerations for GPs. In public markets, active management is associated with well-documented performance disparities and opportunity costs (e.g., French 2008; Fama and French 2010). Still, public allocations can be made less manager dependent by holding broad-market portfolios. "Holding the market" can be difficult in private allocations, however, especially for smaller investors. Investors are typically forced to choose among GPs, which can be associated with material search and monitoring costs. In addition, the illiquidity of private allocations means the choice of GP is either binding or costly to revoke through the secondary market. So-called "primary" funds of funds (FoFs) may provide diversification to LPs and "secondary" FoFs may provide liquidity, but Harris, Jenkinson, Kaplan, and Stucke (2018) argue that "Against these advantages must be weighed the additional fees charged by the FoF manager" (p. 287). 16

3.2 Relative performance and the choice of benchmark

Figure 4 plots, for each asset class, weighted averages of KS-PMEs and DAs for the average fund as a function of vintage age. The averaging is across vintages with weights determined by inflation-adjusted total committed capital (in 2022Q4 dollars).

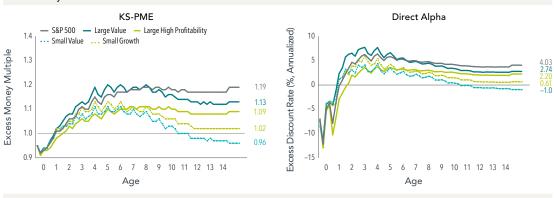
For BOs and VCs (Panels A and B), we use the same benchmarks as in the example in Section 2. The average BO outperforms the S&P 500 Index over its lifetime with a KS-PME of 1.19 and an annualized DA of 4.03%. Using the style benchmarks, KS-PME drops to between 1.13 (against large cap value) and 0.96 (against small cap value), and the corresponding DAs are between 2.74% and –1.0% annualized. Similarly, the average VC has a lifetime KS-PME of 1.15 and a lifetime annualized DA of 4.59% relative to the S&P 500 Index. Using the style benchmarks, KS-PME drops to between 1.10 (against large cap value) and 0.97 (against small cap value), with annualized DAs between 3.92% and 1.09%. To naverage, both measures converge to their long-term values at around 10 years. One potential explanation for their distinctive hump shape is that, early on, GPs exit more successful deals to achieve LPs' hurdle rate, whereas, later on, performance reflects other (average and less successful) deals as well as the impact of carried interest. We reiterate that Figure 4 is for the average fund. While data limitations prevent us from studying dispersion in relative performance, it is plausibly as pronounced as in Figure 3.18

^{16.} Harris et. al (2018) study 294 primary FoFs using PCU data from 1987 to 2007 and find that "FoFs focusing on buyouts [...] underperform direct fund investment strategies in buyout," while "the average performance of FoFs in venture capital is on a par with results from direct venture fund investing" (abstract). They report, for the 190 FoFs for which they have holdings data (their Table 5), that the median number of funds held is 19 for BO-focused FoFs (interquartile range of 14–28) and 26 for VC-focused FoFs (interquartile range of 18–35). More recently, some so-called "evergreen" funds offer exposure to a larger number of GPs at additional fees.

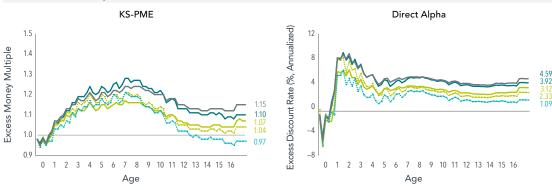
^{17.} Assuming a constant benchmark scaling factor of 1.3 for BOs (Phalippou 2014; Robinson and Sensoy 2016) generally reduced average KS-PME to between 1.11 (against the S&P 500) and 0.87 (against small cap value). Similarly, assuming a constant benchmark scaling factor of 2.0 for VCs (Robinson and Sensoy 2016) generally reduces average KS-PME to between 0.95 (against the S&P 500) and 0.75 (against small cap value).

^{18.} We cannot study the dispersion in KS-PMEs or DAs across funds because, as discussed in Footnote 13, any variable we compute ourselves must be based on average cash flows and NAVs by vintage. Nonetheless, the PCU does in fact report statistics for KS-PMEs using the S&P 500 Index, allowing us to shed some light on dispersion in this case. For BOs, the weighted-average 95th and 5th percentiles are 1.98 and 0.50. For VCs, they are 2.57 and 0.22. In both cases, the range is comparable to that of the corresponding TVPI (see Figure 3). Moreover, the Jensen effect is in this case negligible: The weighted average across vintages of the average KS-PME reported by the PCU is 1.18 for BOs and 1.11 for VCs, nearly identical to the 1.19 and 1.15 we report in Figure 4 for the weighted average KS-PME based on average cash flows and NAVs.

FIGURE 4: Relative Performance for Average Fund
Panel A: Buyout



Panel B: Venture Capital



Panel C: Private Credit



Panel D: Private Real Estate



Past performance is not a guarantee of future results.

In USD. This figure shows weighted averages across vintages of the average fund's lifetime KS-PME and DA as a function of age by asset class and benchmark. The averaging is across vintages with weights determined by inflation-adjusted total committed capital (using US CPI and expressed in 2022Q4 dollars). Age is measured in years relative to the end of Q4 of the vintage year. All cash flows and NAVs are net of management fees and carried interest. DAs over periods shorter than one year are not annualized. See Appendix K for index definitions and Appendix L for disclosures. Data are quarterly, end with 2022, and start in 1980Q1 for venture capital, 1986Q1 for buyouts, and 1993Q1 for private credit and private real estate.

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Table F1 in the Appendix shows test statistics for KS-PMEs and DAs. We compute the average fund's lifetime KS-PME and DA for each vintage, then report the average across vintages along with a Newey and West (1987, 1994) *t*-statistic. We focus on (inflation-adjusted) capitalization-weighted averages across vintages, but the table also shows results for equal-weighted averages. To ensure the results reflect mature funds, we consider vintages with a minimum age of 10 years. For BOs, the average fund significantly outperforms the S&P 500 Index as well as the large cap value benchmark, but its performance is statistically indistinguishable from that of the remaining benchmarks. For VCs, we find no reliable differences in performance relative to any of the five benchmarks.

Panel C of Figure 3 shows results for private credit. We benchmark against the Bloomberg US Credit Index (i.e., investment-grade securities) and US Corporate High Yield Index. The average fund outperforms the credit index with a KS-PME of 1.09 and a DA of 2.64% annualized but slightly underperforms the high-yield index with a KS-PME of 0.97 and DA of -0.55% annualized. Table F1 in the Appendix shows that only the outperformance against the credit index is statistically reliable.¹⁹

Lastly, Panel D of Figure 3 shows results for private real estate, which we benchmark against the Dow Jones US Select REIT Index (a proxy for the performance of public equity real estate investment trusts) and the Fama/French US Real Estate Industry Research Portfolio (a proxy for the performance of public real estate companies not classified as REITs). The average fund's lifetime KS-PME is 0.97 against the industry portfolio and 0.81 against REITs. The corresponding DAs are -0.56% and -4.01% annualized. Table F1 in the Appendix shows that only the underperformance relative to REITs is reliable.²⁰

^{19.} We also compute the average private credit fund's KS-PME and DA relative to the LSTA US Leveraged Loan Index (a proxy for the performance of the secondary market for leveraged loans, starting from 1997), which is popular among practitioners. We find a lifetime KS-PME of 1.05 and a corresponding DA of 2.40% annualized. Repeating the tests in **Table F1** shows that this outperformance is statistically reliable.

^{20.} Assuming a constant benchmark scaling factor of 1.3 for real estate (as for BOs; Footnote 17) decreases the average fund's KS-PME against REITs (from 0.81 to 0.75) but increases it against the industry portfolio (from 0.97 to 0.99). For completeness, we also compute the average real estate fund's KS-PME and DA relative to two private real estate benchmarks: the NCREIF Property Index (a proxy for the unlevered performance of private commercial real estate held for investment purposes) and the NFI-ODCE Index (a proxy for the net performance of open-end diversified core equity private real estate funds). The KS-PME is 0.95 against the NCREIF index and 0.88 against the NFI-ODCE index. The corresponding DAs are -0.73% and -2.21% annualized. Repeating the tests in Table F1 shows that none of these performance differences are reliable.

4. Diversification Benefits

Private funds, by holding unlisted assets, expand the opportunity set for public investors. These funds may therefore offer genuine diversification benefits, provided their returns are not perfectly correlated with listed asset returns. This is what we seek to test in this section.

There are reasonable arguments for and against whether private funds provide *meaningful* diversification benefits. On one hand, listed and unlisted assets are part of the same economy and exposed to the same aggregate shocks, and the underlying companies or projects compete for the same resources and customers. A luxury hotel group acquired through a buyout will likely share a customer base with a listed but otherwise similar hotel group, and both can expect a drop in revenue during a pandemic. Skeptics therefore argue investors should not expect extraordinary diversification benefits from unlisted assets simply because they are unlisted (e.g., Phalippou 2021, chap. 17). On the other hand, proponents argue that listed and unlisted assets may be similar on certain characteristics but sharply differ on others. Successful VC-backed startups may resemble young, small, fast-growing technology stocks, but the highly skewed payoff structure and illiquidity of VC allocations can deliver markedly different return streams (e.g., Cochrane 2005, p. 5).²¹

To calculate correlations in a parsimonious manner, we adopt the factor, or "spanning," regression approach well-known from public markets. Specifically, we determine the extent to which public factor models explain aggregate periodic private fund returns. While also commonly employed for illiquid assets, this approach is not without its issues and there are alternatives (discussed below). The main issue is that periodic fund returns, which depend on interim valuations, can appear nonsynchronous (lagged) and less volatile (smoothed) compared to listed asset returns, leading to understated factor loadings. We adjust for lagging by summing coefficients on lagged factor returns (Scholes and Williams 1977; Dimson 1979; Asness, Krail, and Liew 2001; Ewens, Jones, and Rhodes-Kropf 2013), and we "unsmooth" returns using autoregressive models (Geltner 1991, 1993).

^{21.} Speaking at the 2022 Fiduciary Investors Symposium, Professor Steve Kaplan of the University of Chicago Booth School of Business is quoted as saying the beta of BOs is "less than you think," pointing to the work of Brown, Ghysels, and Gredil (2023). He adds that "the companies they buy aren't necessarily as high beta as your typical company," and "when they do add value to their companies, that value added basically has zero beta." He concludes that "Looking backward, buyout has been an unbelievably good place to be both in terms of say alpha and in terms of diversification," while "Venture has been mixed, but on average pretty good." Source: Ben Hurley, "Private Equity Beta Is Lower than Many Think," Top1000funds.com, June 7, 2022.

4.1 Aggregate quarterly net returns

To construct a time series of aggregate quarterly net returns by asset class, let

$$IRR_{it}^{Q} = \frac{NCF_{it} + NAV_{it}}{NAV_{i,t-1}} - 1$$
(2)

be fund i's quarterly net "return" at the end of quarter t, where NCF $_{it}$ is net cash flow, i.e., distribution minus paid-in capital. Equation (2) coincides with the IRR of a one-period fund (see Footnote 8) and is also the definition used by Ewens et al. (2013). Given a vintage, we get its simple average quarterly net return directly from the PCU. Our aggregate quarterly net return, denoted IRR $_{it}^{Q}$, is then the weighted average across vintages, where the weights are determined by inflation-adjusted total committed capital. Averaging across vintages helps mitigate vintage-specific idiosyncracies. To avoid overinfluence of newly raised funds, whose quarterly returns mechanically reflect capital calls and the buildup of investment positions, vintages are included in the calculation once they pass one year.

To get an unsmoothed version of IRR_{it}^Q , we follow Geltner (1991, 1993) and assume each raw return is a weighted average of the unsmoothed return and lags of raw returns,

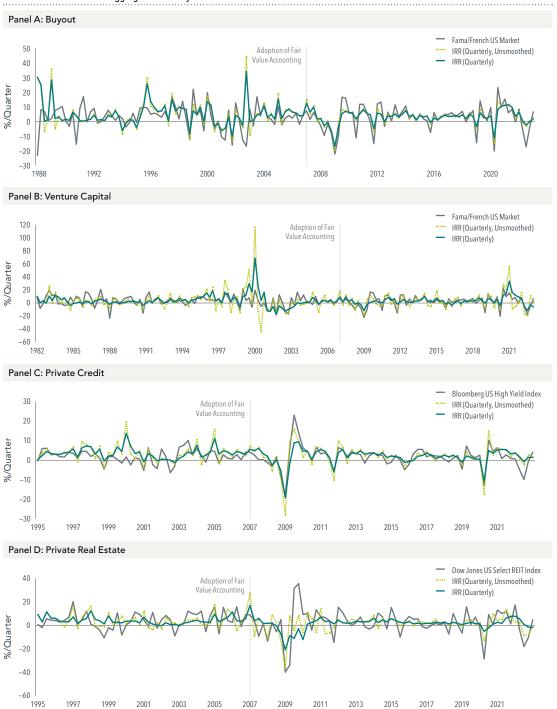
$$\overline{IRR}_{t}^{Q} = \varphi_{0}\overline{IRR}_{t}^{Q,Unsm} + \sum_{l=1}^{L} \varphi_{l}\overline{IRR}_{t-l}^{Q}, \tag{3}$$

with $\sum_{l=0}^L \varphi_l = 1$. Assuming $\overline{\text{IRR}}_t^{Q,\text{Unsm}} = \mu + \eta_t$, where the $\{\eta_t\}$ are independent and identically distributed with mean zero, **Equation (3)** is an autoregressive (AR) model of order L. Hence, unsmoothed returns are given by $\overline{\text{IRR}}_t^{Q,\text{Unsm}} = \hat{\mu} + \hat{\epsilon}_t/(1 - \sum_{l=1}^L \hat{\varphi}_l)$, where $\hat{\mu}$ is the mean of $\overline{\text{IRR}}_t^Q$, $\hat{\varphi}_l$, ..., $\hat{\varphi}_L$ are the estimated AR coefficients, and $\hat{\epsilon}_t = \hat{\varphi}_0 \hat{\eta}_t$ is the model residual for quarter t. We choose the AR order, L, as the maximum lag for which the partial autocorrelation is reliably nonzero, allowing for interim autocorrelations to be unreliable.

Figure 5 shows time-series plots of quarterly returns by asset class along with the returns to our proxies for the comparable public market: the Fama/French US Market for BOs and VCs, the Bloomberg US Corporate High Yield Index for credit, and the Dow Jones US Select REIT Index for real estate.²² There is visible comovement with the market proxies, especially after the adoption of fair value accounting in November 2007. Unsmoothing has noticeable effects, particularly for real estate. The 2008–2010 crisis years feature notable drawdowns and subsequent recoveries, which look remarkably similar for the public and private return series. We will return to the effects of the crisis years on our results.

^{22.} Burgiss and Cambridge Associates publish popular indices of aggregate periodic private fund returns. Both use definitions akin to Equation (2) for fund-level periodic returns, but neither is explicit about the aggregation across funds. Kortweg and Westerfield (2022, sec. 3.2) and Korteweg (2023, sec. 3.4), however, argue that these indices are effectively NAV-weighted. We prefer (inflation-adjusted) capitalization-weighted indices because they reflect the economic size of the entities and avoid a second source of lagging and smoothing. Nonetheless, because each capitalization-weighted series plotted in Figure 5 has a time-series correlation of at least 0.9 with its NAV-weighted version, the choice of weighting scheme should not have material impact.

FIGURE 5: Time-Series of Aggregate Quarterly Net Returns



Past performance is not a guarantee of future results.

In USD. This figure shows weighted average across vintages of the average fund's quarterly net return, both raw and unsmoothed, as a function of calendar quarter by asset class. The weights are determined by inflation-adjusted total committed capital (using US CPI and expressed in 2022Q4 dollars). The figure also shows the quarterly returns to our proxies for the relevant public markets. See **Appendix K** for index definitions and **Appendix L** for disclosures. Unsmoothed returns are based on an AR model with order 1 for BOs, 4 for VCs, 1 for private credit, and 6 for private real estate. To avoid overinfluence of newly raised funds, vintages are included in the calculation once they pass one year of age. Data are quarterly, through 2022Q4, and start in 1981Q4 for venture capital, 1987Q4 for buyout, and 1994Q4 for private credit and private real estate

4.2 Factor regressions: intuition and interpretation

Factor regressions quantify the extent to which a test asset's returns are explained by a linear factor model. Formally, if the asset generates "abnormal" returns relative to the model, it improves upon the ex post mean-variance efficient portfolio that optimally combines the risk-free asset and the model factors over the sample period (e.g., Jensen 1968; Huberman and Kandel 1987; Gibbons, Ross, and Shanken 1989; Fama 1998; Barrilas and Shanken 2017; Fama and French 2018; Detzel, Novy-Marx, and Velikov 2023). That is, a hypothetical investor holding the "tangency" portfolio implied by the model factors would have seen an improved in-sample mean-variance tradeoff by adding the asset. Conversely, if the asset is spanned by the model factors, it is redundant over the sample period.

Fama and French (2018) consider the case where the test asset is a single factor: "[T]he increase in the max squared Sharpe ratio for a model's factors when [factor] i is added to the model is a_i^2/σ_i^2 ," where a_i is the intercept and σ_i is the residual standard error, so that "the factor's marginal contribution [...] is small if the factor's expected return is explained well by other factors (a_i is close to zero) and/or its variation not explained by other factors (σ_i) is large"(p. 5).²³ Since a_i/σ_i is proportional to the t-statistic for the null of a zero intercept, the standard criterion for adding factor i to a model is that a_i is reliably different from zero.

Our focus, however, is not on whether private fund returns constitute nonredundant factors. Rather, it is on the extent to which adding exposure to private funds would have implied meaningful diversification benefits for public investors. As such, we adopt broader criteria beyond the statistical reliability of a_i . Specifically, we also consider a_i 's economic significance, and we put considerable emphasis on the explained variation, measured by the regression's (adjusted) R^2 . The latter is particularly useful for our purpose because it is the squared correlation between the test asset and the model's replicating strategy given the factor exposures. An R^2 close to 1 along with a near-zero a_i suggest limited benefits from adding the asset over the period.²⁴

^{23.} Fama and French (2018) comment on the relation $(a_i/\sigma_i)^2 = SR^2(\{f,i\}) - SR^2(f)$, where $SR^2(\cdot)$ is the maximum squared Sharpe ratio attainable from a set of assets, i is the test asset, and f is a set of factors. The multivariate version of this relation is due to Gibbons, Ross, and Shanken (1989) and underlies their (GRS) F-test statistic for the null of jointly zero intercepts. Barrilas and Shanken (2017) propose $SR^2(f)$ as a model-comparison criterion. They show that the best-performing model, in terms of the ability to explain the average returns to test assets, is the one with the highest $SR^2(f)$, irrespective of the test assets. Fama and French (2018) use $SR^2(f)$ to compare nested and unnested versions of the Fama and French (2015) five-factor model. Detzel, Novy-Marx, and Velikov (2023) use it to compare various factor models after accounting for estimated transaction costs and conclude that "Accounting for transaction costs, the Fama and French (2015, 2018) five-factor model has a significantly higher squared Sharpe ratio than either of these alternative models" (abstract).

^{24.} To illustrate, consider regressions of the emerging markets (EM) factors from Ken French's website using the developed markets (DM) five-factor model from 1990 to 2022. For the EM market factor, the adjusted R^2 is 67% and the intercept is 16 bps/month (t = 0.83). One third of the variation remains unexplained, and, though statistically unreliable, the average unexplained return is economically sizeable, suggesting considerable diversification benefits over the period. For the EM size factor, the intercept is just 5 bps/month (t = 0.39) but the explained variation is a mere 10%, suggesting highly dissimilar return variation. For the EM value, profitability, and investment factors, the explained variation is 7–24% and the intercept is 23–46 bps/month (t-statistics of 2.39–3.99), indicating clear diversification benefits. Sinquefield (1996) also finds that non-US style portfolios are better diversifiers for US stocks than are non-US market portfolios.

Note that it is tempting to view a_i as a measure of private funds' relative performance, similar to KS-PME and DA. We caution, however, against this interpretation. a_i is a function of aggregate quarterly fund returns and estimated factor loadings. Because the quarterly returns rely on NAVs, they cannot be compounded to reflect annualized or lifetime performance, and the same is true for a_i . Moreover, because factor loadings are estimated with noise and may change over time, it can be difficult or require long time series to pin down a_i . KS-PME and DA, on the other hand, can be calculated without estimating parameters and are meaningful even for a single fund. For these reasons, we interpret a_i as the average unexplained aggregate quarterly return or, simply, the average model error.

4.3 Factor regressions for BOs

Table 2 shows factor regressions for BOs. Panel A reports summary statistics for quarterly net excess returns (i.e., net of fees and in excess of the quarterly returns to one-month T-bills) before and after the adoption of fair value accounting in 2008Q1. We unsmooth returns using an AR(1) in either sample. BOs earn reliably positive average net excess returns, but smoothing understates their volatility. Adjusting for heteroskedasticity and autocorrelation (Newey and West 1987, 1994) yields similar volatilities for raw and unsmoothed returns, which is comparable to the volatility observed for market returns.

Panel B shows regressions on the Fama and French (2015) factors. Specifications one through four are through 2007Q4, while specifications five through eight are post-2007. All t-statistics use Newey-West standard errors. The first specification shows that, in the sample ending with 2007Q4, a simple market model implies an unreliable beta of just 0.14 (t = 0.69) and only 0.7% explained variation, leaving a large average unexplained return (3.84%/quarter, t = 3.24). The second specification adds three lags of market excess returns. As is standard in the literature, we choose the lag-length by successively adding lagged returns until we no longer see reliable coefficients, allowing the interim coefficients to be unreliable. Summing the coefficients increases beta to 0.51 (t = 2.12).²⁵ Still, R^2 is only 10%, and the intercept remains large (3.11%/quarter, t = 2.95). The third specification adds the remaining factors' contemporaneous and lagged returns. Following Ewens et. al (2013), we use the same number of lags for all factors, i.e., three.²⁶ This has limited impact, however, because the additional loadings are unreliable. The fourth specification shows that using unsmoothed returns increases R^2 to 15% but has otherwise little impact.

^{25.} We compute the t-statistic for the null of a zero sum as $t_{\text{sum}} = \mathbf{1}' \widehat{\boldsymbol{\beta}} / (\mathbf{1}' \widehat{\boldsymbol{\Sigma}}_{NW} \mathbf{1})^{1/2}$, where $\widehat{\boldsymbol{\beta}} = (\hat{\beta}_0, \hat{\beta}_1, ..., \hat{\beta}_L)'$ is the (L+1)-vector of coefficient estimates for the contemporaneous and lagged variables, $\widehat{\boldsymbol{\Sigma}}_{NW}$ is the estimates' $(L+1) \times (L+1)$ Newy-West variance-covariance matrix, and $\mathbf{1}$ is an (L+1)-vector of ones.

^{26.} A maximum of 3–4 lags is common in the literature (e.g., Asness, Krail, and Liew 2001; Cao, Chen, Liang, and Lo 2013; Ewens, Jones, and Rhodes-Kropf 2013; Boyer, Nadauld, Vorkink, Weisback 2023). In **Table 2**'s specifications (3) and (4), using lags 0–3 means estimating 21 parameters using 81 observations, or 60 degrees of freedom. Adding more lags for some or all the factors risks overfitting the model and rendering it unreliable.

TABLE 2: Factor Regressions for BOs

Panel A: Summary Statistics for Quarterly Net Excess Returns

1987Q4-2007Q4

	Raw	Unsmoothed	Fama/French US Market
Mean Excess Return (%/Quarter)	4.06	4.06	1.69
Volatility (%, ann.)	15.80	17.85	16.30
Volatility (%, Newey-West)	17.92	18.14	15.17
t-Statistic	4.63	4.09	1.86
t-Statistic (Newey-West)	4.08	4.03	2.00

Raw	Unsmoothed	Fama/French US Market
2.62	2.61	2.41
9.74	14.54	18.47
12.94	14.51	18.73
4.16	2.78	2.02
3.13	2.79	1.99

Panel B: Time-Series Regressions of Quarterly Net Excess Returns (Newey-West *t*-statistic in parentheses; reliable estimates in bold)

1987Q4-2007Q4

2008Q1-2022Q4

	(1) Raw Lag 0	(2) Raw Lags 0-3	(3) Raw Lags 0-3	(4) Unsmoothed Lags 0–3	(5) Raw Lag 0	(6) Raw Lags 0-3	(7) Raw Lags 0-3	(8) Unsmoothed Lags 0-3
Intercept (%/Quarter)	3.84 (3.24)	3.11 (2.95)	3.20 (2.51)	3.04 (2.43)	1.53 (2.92)	0.75 (2.09)	0.31 (0.89)	0.19 (0.54)
$R_M - R_F$	0.14 (0.69)	0.51 (2.12)	0.49 (1.58)	0.56 (1.81)	0.45 (7.82)	0.77 (9.61)	0.79 (9.17)	0.78 (11.43)
SMB			0.19 (0.57)	0.30 (0.79)			0.16 (1.30)	0.29 (3.91)
HML			-0.12 (-0.35)	-0.20 (-0.59)			-0.14 (-1.34)	-0.32 (-2.06)
RMW			0.06 (0.12)	0.18 (0.36)			0.20 (1.62)	0.11 (1.42)
CMA			-0.14 (-0.31)	-0.17 (-0.35)			0.30 (1.87)	0.50 (1.94)
Adjusted R ² (%)	0.7	10.1	13.8	15.1	73.2	85.7	84.6	79.8
CI (Adjusted R ² , 95%)	(-1.3, 19.5)	(-3.2, 17.8)	(-7.1, 37.4)	(-5.1, 39.7)	(58.6, 82.2)	(78.3, 89.9)	(78.3, 89.4)	(65.7, 81.0)
N (Quarters)	81	81	81	81	60	60	60	60
Degrees of Freedom	79	76	60	60	58	55	39	39

Past performance is not a guarantee of future results.

In USD. Panel A shows summary statistics for aggregate quarterly net excess BO returns and quarterly Fama/French market excess returns before the adoption of fair value accounting (198704–200704) and after (200801–202204). Unsmoothed returns are based on an AR(1) model in either sample, determined using the partial autocorrelation function. Panel B shows time-series regressions on the quarterly returns to the Fama and French (2015) factors with Newey-West t-statistics for the null of zero in brackets and estimates with It| ≥ 2 indicated in bold. Quarterly factor returns are the differences between the compound returns to each of the long and short sides. The regressions correct for lagging by summing the coefficients on lagged regressors, where the number of lags is chosen based on the significance of the coefficients on lagged market excess returns. Test-statistics for the summed coefficients are based on the estimated Newey-West variance-covariance matrix. All Newey-West adjustments use automatic lag selection. Each 95% confidence interval for adjusted R² uses the bias-corrected and accelerated method on 10,000 nonparametric bootstrap samples. See Appendix L for disclosures.

In line with our results through 2007Q4, Ewens et. al (2013) find that BOs in aggregate have a beta of 0.7–0.8, negligible size and value factor loadings, and around 35% explained variation. The relatively low beta is perhaps most surprising given the levered nature of BO investments, but it is a common finding in the literature and robust across different data sets and methodologies. (Appendix G.1 provides an overview of BO factor exposures in the literature.) Axelson, Sørensen, and Strömberg (2014) argue that the simple Modigliani-Miller reasoning that suggests a beta well above 1 for BOs only applies to gross returns, whereas carried interest can materially reduce the beta of net returns. Intuitively, carried interest drags down net returns in up markets but reduces the gap to gross returns in down markets, dampening the sensitivity to market movements. They estimate a gross-return beta around 2.3 using deal-level BO data from a large LP (see Kortweg 2023, sec. 6, for more on the effects of carried interest on beta).

Panel B's post-2007 regressions show a dramatic increase in explanatory power. The baseline market model's R^2 is 73%, though beta remains just 0.45 (t = 7.82), leaving a considerable intercept (1.53%/quarter, t = 2.92). Adding lags increases beta to 0.77 (t = 9.61), raises R^2 to 86%, and halves the intercept (0.75%/quarter, t = 2.50). Controlling for the remaining factors has little impact on R^2 , though this reveals positive but noisy profitability and investment factor loadings (t-statistics of 1.62 and 1.87) and further halves the intercept (0.31%/quarter, t = 0.89). Using unsmoothed returns, the explained variation drops slightly to 80%, but so does the intercept (0.19%/quarter t = 0.54, and there is now a reliably positive size loading, a reliable negative value loading, and a positive but noisy investment factor loading (t = 1.94). Despite the relatively high point estimate for R^2 in the last specification, its 95% confidence interval suggests we cannot reject that 19%–34% of the return variation remains unexplained.²⁷ Table I1 in the Appendix shows that the higher explained variation we find post-2007 for BOs is not driven by the 2008–2010 crisis years (but that the intercepts are higher when excluding the crisis years).

In sum, the inability of public market factors to explain aggregate BO returns found here and elsewhere in the literature appears to be driven by the period predating the adoption of fair value accounting. Post-2007, the explanatory power is dramatically higher, and the factor loadings suggest comovement with relatively smaller stocks that have high valuations and low investment. Nonetheless, we cannot reject that up to one-third of the variation in aggregate BO returns remains unexplained by these exposures. Since BOs expand public investors' opportunity set, our results suggest they have still managed to provide meaningful diversification benefits over this latter period.

4.4 Factor regressions for VCs

Table 3 repeats the exercise for VCs. We unsmooth their returns using an AR(4) over the sample ending with 2007Q4 and an AR(3) post-2007. Panel A shows that VCs in aggregate earn reliably positive average net excess returns over the latter period, but not the earlier one. They are nearly twice as volatile as the market over the sample ending with 2007Q4 but have market-like volatility post-2007.

In Panel B's early-sample regressions, adding four lags to the market model implies a beta of 1.42 (t = 3.78), 31% explained variation, and an unreliable intercept (-0.24/quarter, t = -0.30). Interestingly, controlling for the remaining factors increases R^2 to 58% due to a large, reliably negative profitability loading (-1.47, t = -3.47), but it also reduces beta to 0.55 (t = 2.23) and implies a very large intercept (4.56%/quarter, t = 2.89). Using unsmoothed returns moderates R^2 to 44%, raises beta back to 1.00 (t = 3.30), and renders the negative profitability loading unreliable, resulting in an economically significant but statistically unreliable average unexplained return (2.49%/quarter, t = 1.63). While unreliable, the negative value loadings found here are broadly consistent with the literature (see Appendix G.2).

Similar to BOs, the post-2007 VC results show generally greater explained variation. When adjusted for lagging, the five-factor model produces a beta of 0.82 (t=5.41), a reliably positive size loading, a reliably negative value loading, and a positive, marginally significant profitability loading. This explains 70% of the variation and leaves an intercept of just 5 basis points (bps)/ quarter. Using unsmoothed returns moderates the explained variation to 65% and implies generally noisier factor loadings, resulting in a larger though still statistically unreliable intercept (0.24%/quarter, t=0.25). The 95% confidence interval for R^2 suggests we cannot reject that 30% to 56% of the variation remains unexplained. Table I1 in the Appendix shows very similar results when excluding the 2008–2010 crisis years.

Overall, public market factors have considerable explanatory power for aggregate VC returns, especially post-2007, after the adoption of fair value accounting. Over this period, VCs appear to comove with small cap growth stocks, though we cannot reject that up to half of the return variation remains unexplained by these exposures. This suggests VCs continued to deliver material diversification benefits for public investors post-2007.

TABLE 3: Factor Regressions for VCs

Panel A: Summary Statistics for Quarterly Net Excess Returns

1981Q4-2007Q4

2008Q1-2022Q4

	Raw	Unsmoothed	Fama/French US Market
Mean Excess Return (%/Quarter)	2.59	2.62	2.10
Volatility (%, ann.)	18.99	33.04	16.43
Volatility (%, Newey-West)	31.94	32.56	14.69
t-Statistic	2.79	1.62	2.62
t-Statistic (Newey-West)	1.66	1.65	2.93

Raw	Unsmoothed	Fama/French US Market
2.91	2.80	2.41
13.22	21.91	18.47
20.43	21.70	18.73
3.41	1.98	2.02
2.21	2.00	1.99

Panel B: Time-Series Regressions of Quarterly Net Excess Returns (Newey-West t-statistic in parentheses; reliable estimates in bold)

1981Q4-2007Q4

2008Q1-2022Q4

	(1) Raw Lag 0	(2) Raw Lags 0-4	(3) Raw Lags 0-4	(4) Unsmoothed Lags 0-4	(5) Raw Lag 0	(6) Raw Lags 0-2	(7) Raw Lags 0-2	(8) Unsmoothed Lags 0-2
Intercept (%/Quarter)	1.56 (1.27)	-0.24 (-0.30)	4.56 (2.89)	2.49 (1.63)	1.82 (2.68)	0.94 (1.66)	0.05 (0.09)	0.24 (0.25)
$R_M - R_F$	0.49 (3.19)	1.42 (3.78)	0.55 (2.53)	1.00 (3.30)	0.45 (4.68)	0.83 (3.57)	0.82 (5.41)	0.78 (3.36)
SMB			-0.02 (-0.09)	-0.45 (-1.03)			0.85 (3.08)	0.74 (2.17)
HML			-0.36 (-1.02)	-0.52 (-1.17)			-0.50 (-2.69)	-0.62 (-1.83)
RMW			-1.47 (-3.47)	-0.62 (-1.16)			0.73 (1.83)	0.70 (1.17)
CMA			-0.65 (-1.26)	-0.24 (-0.35)			-0.31 (-1.05)	-0.44 (-1.05)
Adjusted R ² (%)	17.2	30.6	58.3	43.5	38.4	53.3	70.5	65.2
CI (Adjusted R ² , 95%)	(4.8, 29.5)	(13.0, 39.0)	(39.1, 62.0)	(19.3, 48.2)	(21.0, 56.5)	(34.5, 63.9)	(44.2, 76.4)	(43.6, 69.7)
N (Quarters)	105	105	105	105	60	60	60	60
Degrees of Freedom	103	99	79	79	58	56	44	44

Past performance is not a guarantee of future results.

In USD. Panel A shows summary statistics for aggregate quarterly net excess VC returns and quarterly Fama/French market excess returns before the adoption of fair value accounting (198104-200704) and after (200801-202204). Unsmoothed returns are based on an AR(4) before the adoption of fair value accounting and an AR(3) after, determined using the partial autocorrelation function. Panel B shows time-series regressions on the quarterly returns to the Fama and French (2015) factors with Newey-West £-statistics for the null of zero in brackets and estimates with $|t| \ge 2$ Indicated in bold. Quarterly factor returns are the differences between the compound returns to the long and short sides. The regressions correct for lagging by summing the coefficients on lagged regressors, where the number of lags is chosen based on the significance of the coefficients on lagged market excess returns. Test-statistics for the summed coefficients are based on the estimated Newey-West variance-covariance matrix. All Newey-West adjustments use automatic lag selection. Each 95% confidence interval for adjusted R^2 uses the bias-corrected and accelerated method on 10,000 nonparametric bootstrap samples. See **Appendix** £ for disclosures.

4.5 Factor regressions for private credit

Table 4 shows factor regressions for private credit. We use an AR(1) to unsmooth their returns through 2007Q4 and post-2007. Panel A shows that private credit funds in aggregate earn reliably positive average net excess returns over either period. The average return is over three times that of the high-yield index in the earlier sample but is more in line with it post-2007. The two series have similar volatilities irrespective of period.

Panel B shows regression results for a two-factor model. The first factor is the excess return to the high-yield index. We take this as the proxy for the market factor within high-yield bonds, but it will naturally also capture the credit premium earned by high-yield bonds relative to T-bills.

The second factor is a proxy for the term premium within the high-yield market, defined as the return spread between the "Long" and "Intermediate" subindices of the Bloomberg US Corporate High Yield Index.²⁸ These are akin to the two bond factors considered by Fama and French (1993), but with a focus on the high-yield market.

TABLE 4: Factor Regressions for Private Credit

Panel A: Summary Statistics for Quarterly Net Excess Returns

199404-200704

2008Q1-2022Q4

	Raw	Unsmoothed	Bloomberg US High Yield
Mean Excess Return (%/Quarter)	2.70	2.69	0.85
Volatility (%, ann.)	5.55	7.88	6.88
Volatility (%, Newey-West)	7.23	7.49	8.31
t-Statistic	7.09	4.98	1.80
t-Statistic (Newey-West)	5.44	5.23	1.50

Raw	Unsmoothed	Bloomberg US High Yield
1.55	1.55	1.51
8.34	11.93	11.82
9.68	10.58	12.38
2.89	2.02	1.98
2.49	2.27	1.89

Panel B: Time-Series Regressions of Quarterly Net Excess Returns (Newey-West t-statistic in parentheses; reliable estimates in bold)

1994Q4-2007Q4

200801	-202204

	(1) Raw Lag 0	(2) Raw Lags 0-1	(3) Raw Lags 0-1	(4) Unsmoothed Lags 0–1
Intercept (%/Quarter)	2.48 (4.92)	2.38 (4.31)	2.64 (4.74)	2.57 (4.19)
$R_{HY} - R_F$	0.27 (3.32)	0.37 (2.37)	0.39 (3.12)	0.56 (4.83)
$R_{HY; \ Long} - R_{HY; \ Intem.}$			-0.27 (-2.47)	-0.34 (-2.60)
Adjusted R ² (%)	9.1	9.7	10.3	16.6
CI (Adjusted R ² , 95%)	(-1.7, 29.9)	(-3.6, 30.9)	(-6.5, 28.9)	(-3.0, 36.9)
N (Quarters)	53	53	53	53
Degrees of Freedom	51	50	48	48

(5) Raw Lag 0	(6) Raw Lags 0-3	(7) Raw Lags 0-3	(8) Unsmoothed Lags 0–3
0.69 (1.31)	0.24 (0.45)	0.35 (0.80)	0.10 (0.21)
0.57 (9.84)	0.88 (4.71)	0.89 (4.89)	0.93 (4.41)
		-0.13 (-0.62)	0.00 (0.01)
65.3	72.5	77.5	81.5
(40.7, 80.2)	(55.2, 84.6)	(59.9, 87.6)	(61.9, 88.3)
60	60	60	60
58	55	51	51

Past performance is not a guarantee of future results.

In USD. Panel A shows summary statistics for aggregate quarterly net excess returns to private credit funds and quarterly excess returns to the Bloomberg US High Yield Index before the adoption of fair value accounting (1994Q4–2022Q4) and after (2008Q1–2022Q4). Unsmoothed returns are based on an AR(1) in either sample, determined using the partial autocorrelation function. Panel B shows time-series regressions using a two-factor model that uses the high-yield index as the market and includes a proxy for the term premium within the high-yield market (the return spread between the "long" and "Intermediate" sub-indices of the Bloomberg US High Yield Index). Panel B also shows Newey-West t-statistics for the null of zero in brackets and estimates with $|t| \ge 2$ indicated in bold. The regressions correct for lagging by summing the coefficients on lagged regressors, where the number of lags is chosen based on the significance of the coefficients on lagged market excess returns. Test-statistics for the summed coefficients are based on the estimated Newey-West variance-covariance matrix. All Newey-West adjustments use automatic lag selection Each 95% confidence interval for adjusted R^2 uses the bias-corrected and accelerated method on 10,000 nonparametric bootstrap samples. See **Appendix L** for disclosures.

Over the early sample, the lag-adjusted market model implies a beta of 0.37 (t = 2.37), just 10% explained variation, and a large intercept (2.38%/quarter, t = 4.31). Controlling for the term premium reveals a reliably negative term loading but has otherwise limited impact on the fit. Using unsmoothed returns further increases beta to 0.56 (t = 4.83) and the explained variation to 17%, but the intercept remains large (2.57%/quarter, t = 4.19).

Post-2007, the two-factor model yields a beta of 0.89 (t = 4.89), a negative but unreliable term loading, a substantially higher R^2 of 77%, and a sizable but unreliable intercept (0.35%/quarter,

^{28.} From 1994Q4 to 2022Q4, the excess return to the high-yield index averaged 1.20%/quarter with a Newey-West t-statistic of 2.33 (Table 4, Panel A). Over the same period, our proxy for the term premium within the high-yield market averaged 1.10%/quarter with a t-statistic of 4.13. Regressing on it the high-yield index's excess return implies an intercept of 0.74%/quarter with a t-statistic of 2.22 and an R^2 of 28%. We do not include a separate proxy for the credit premium within the high-yield market (e.g., the return spread between B rated and Ba/BB rated bonds) because such proxies tend to be redundant in the two-factor model that includes the high-yield market's excess return and the term premium within the high-yield market.

t=0.80). Using unsmoothed returns increases beta to 0.93 (t=7.41), reduces the term loading to zero, and further increases R^2 to 82%, resulting in an average unexplained return of just 10 bps/quarter (t=0.21). Despite the high point estimate for R^2 in the last specification, we cannot reject that 12% to 38% of the variation is left unexplained by the factor exposures. Table I1 in the Appendix shows that excluding the 2008–2010 crisis years in the last specification implies only slightly lower explained variation.

In sum, the high-yield bond market has considerable explanatory power for aggregate private credit returns, especially post-2007. Still, we cannot reject that up to one-third of the return variation remains unexplained over this period, suggesting private credit funds in aggregate continued to provide meaningful diversification benefits for public investors.

TABLE 5: Factor Regressions for Private Real Estate

Panel A: Summary Statistics for Quarterly Net Excess Returns

199404-200704

2008Q1-2	2022Q4
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	Raw	Unsmoothed	Dow Jones US Select REIT Index
Mean Excess Return (%/Quarter)	3.28	3.28	2.47
Volatility (%, ann.)	6.60	12.12	14.82
Volatility (%, Newey-West)	7.44	12.36	16.25
t-Statistic	7.23	3.93	2.43
t-Statistic (Newey-West)	6.41	3.86	2.21

Raw	Unsmoothed	Dow Jones US Select REIT Index
0.94	0.90	2.03
9.70	18.59	24.92
17.36	17.02	17.60
1.49	0.75	1.26
0.84	0.82	1.79

Panel B: Time-Series Regressions of Quarterly Net Excess Returns (Newey-West *t*-statistic in parentheses; reliable estimates in bold)

1994Q4-2007Q4

2008Q1-2022Q4

	(1) Raw Lag 0	(2) Raw Lags 0-9	(3) Unsmoothed Lags 0-9	(4) Raw Lag 0	(5) Raw Lags 0-5	(6) Unsmoothed Lags 0-5
Intercept (%/Quarter)	3.02 (5.03)	1.76 (4.03)	0.53 (0.98)	0.66 (0.59)	-0.58 (-0.81)	-0.26 (-0.31)
$R_{REIT} - R_F$	0.11 (4.22)	0.57 (4.23)	1.03 (4.49)	0.14 (3.41)	0.76 (4.52)	0.54 (4.64)
Adjusted R ² (%)	3.8	4.1	14.4	10.6	51.8	34.7
CI (Adjusted R ² , 95%)	(-1.8, 16.1)	(-15.2, 10.3)	(-13.3, 26.9)	(-1.7, 53.4)	(21.8, 61.4)	(-3.5, 52.9)
N (Quarters)	53	53	53	58	53	53
Degrees of Freedom	51	42	42	0.84	0.82	1.79

Past performance is not a guarantee of future results.

In USD. Panel Panel A shows summary statistics for aggregate quarterly net excess returns to private real estate funds and quarterly excess returns to the Dow Jones U.S. Select REIT Index before the adoption of fair value accounting (1994Q4-2022Q4) and after (2008Q1-2022Q4). Unsmoothed returns are based on an AR(6) before the adoption of fair value accounting and an AR(3) after, determined using the partial autocorrelation function. Panel B shows time-series regressions on the excess returns to the Dow Jones Select REIT Index with Newey-West t-statistics for the null of zero in brackets and estimates with $|t| \ge 2$ indicated in bold. The regressions correct for lagging by summing the coefficients on lagged regressors, where the number of lags is chosen based on the societies on lagged market excess returns. Test-statistics for the summed coefficients are based on the estimated Newey-West variance-covariance matrix. All Newey-West adjustments use automatic lag selection. Each 95% confidence interval for adjusted R^2 usses the bias-corrected and accelerated method on 10,000 nonparametric bootstrap samples. See **Appendix** for disclosures.

4.6 Factor regressions for private real estate

Table 5 shows factor regressions for private real estate. We unsmooth their returns using an AR(6) in the sample through 2007Q4 and an AR(3) post-2007. Panel A shows that real estate funds earn reliably positive average net excess returns over the early sample. Post-2007, the average return is half of that earned by REITs, yet equally volatile, and unreliable.

Panel B shows regression results from a market model with the REIT index as the market (we discuss other models below). Over the early sample, adjusting for lagging implies a beta of 0.57 (t = 4.23), 4% explained variation, and a large intercept (1.76%/quarter, t = 4.03). Using unsmoothed returns nearly doubles beta to 1.03 (t = 4.49) and shrinks the intercept to a sizeable but unreliable 0.53%/quarter (t = 0.98), though the R^2 remains a mere 14%.

Post-2007, the lag-adjusted market model implies a beta of 0.76 (t=4.52), 52% explained variation, and a negative but unreliable intercept (-0.58%/quarter, t=-0.81). Using unsmoothed returns moderates beta to 0.54 (t=4.64) and R^2 to 35%, but the average unexplained return is also cut in half (-0.26%/quarter, t=-0.31). In the last specification, we cannot reject that at most 53% of the return variation is explained by the exposure to REITs. Table I1 in the Appendix shows that excluding the 2008–2010 crisis years in the last specification has little impact on the explained variation but yields a larger intercept. Differences in sector composition is a potential explanation for the relatively low correlations between private real estate funds and REITs.

Table H1 in the Appendix shows regressions of private real estate returns on the Fama and French (2015) factors. Over the sample through 2007, the lag-adjusted model explains just 6% of the variation and leaves a large, reliable intercept. Using unsmoothed returns has limited impact on the fit. Post-2007, adjusting for both lagging and smoothing yields a beta close to 1, 31% explained variation, and a large, *negative* intercept (-1.87%/quarter, t = -2.59). Table I1 shows that excluding the crisis years has little impact on the explained variation but implies a reliably positive intercept relative to REITs.

Overall, private real estate funds in aggregate exhibit only mild return correlations with REITs and public equity factors. This is true even after the adoption of fair value accounting and especially when measured using unsmoothed returns. While their average returns are more modest post-2007, in large part due to the crisis years, the relatively unique return variation suggests a considerable ability to diversify listed real estate.

4.7 Publicly listed private management companies

Lastly, we consider whether the stocks of listed private management companies provide meaningful exposure to aggregate private fund returns. A priori, any such exposure may differ materially from that seen by LPs since, as Korteweg (2023) argues, "listed management companies [...] provide investors the opportunity to share in the fees and carried interest income earned by GPs" (p. 73).

Table J1 in the Appendix shows time-series regressions of the LPX America Listed Private Equity Index on the four unsmoothed aggregate return series plotted in Figure 5. The table shows that while the LPX index is correlated with each of the aggregate private fund return series, these correlations are in general subsumed by the exposures of the LPX index to public equity factors. As such, the stocks of listed management companies appear more closely correlated with public markets than with the underlying private funds.

5. Conclusion

We document wide dispersion in private funds' lifetime performance in all major asset classes: buyout, venture, credit, and real estate. This is true using either total value to paid-in capital (TVPI) or internal rate of return (IRR). In addition, whether private funds on average deliver a premium relative to comparable public investments depends crucially on the choice of benchmark. This is also true in all asset classes and holds using either Kaplan and Schoar's (2005) public market equivalent (KS-PME) or Gredil, Griffith, and Stucke's (2023) direct alpha (DA). Lastly, private funds in aggregate exhibit markedly higher return correlations with public market factors after the adoption of fair value accounting in 2007. Nonetheless, we find a considerable fraction of their return variation remains unexplained post-2007 in all asset classes. Since unlisted assets expand the opportunity set for public investors, this suggests private funds have continued to offer considerable diversification benefits. Our results are based on a comprehensive data set of 6,000 North American private funds from 1980 to 2022 from the MSCI Private Capital Universe (PCU).

We conclude with a brief discussion of our findings, their implications, and potential avenues for future research. The wide performance dispersion reiterates the importance of due diligence when selecting a private fund manager, not least because of the lack of evidence for performance persistence among managers (e.g., Harris, Jenkinson, Kaplan, and Stucke 2023). Investors should question how a manager identifies investment opportunities, the expected sources of added value, the structure of carried interest, how the valuation methodology incorporates mark-to-market considerations, and potential exit strategies.

The sensitivity of relative performance to the choice of benchmark has implications for both investors and academics. For investors, it means any single comparator will likely give an incomplete assessment of opportunity cost. Instead, our results suggest comparing to multiple benchmarks, including style benchmarks that deviate from broad market exposure, in conjunction with the KS-PME and DA measures. The latter are easy to calculate, formally justified by valuation theory, robust to leverage, and valid regardless of the underlying beta or its time variation. For academics, the benchmark sensitivity of relative performance further complicates the study of the so-called *illiquidity premium*, i.e., the supposed extra return demanded by LPs for holding unlisted assets relative to listed ones, all else being equal. Pinning down this premium is difficult (see Kortweg 2023, sec. 3.7.3), and our results suggest caution in interpreting outperformance relative to a given benchmark as evidence for its existence.²⁹

^{29.} Kortweg (2023) argues that "the size of the illiquidity premium is a priori unclear" because "On the one hand, most investors in PE have long horizons and should be well positioned to handle liquidity shocks" while "On the other hand, not only is there illiquidity in the realization of distributions, LPs are also exposed to funding liquidity shocks due to GPs having the discretion to call capital at any time [...] and penalties for defaulting on capital calls are stiff" (p. 65). Others argue that LPs may be willing to pay a premium or forgo some returns for unlisted investment because illiquidity may help them stay disciplined, adopt a long-term view, and avoid overreactions (e.g., Welch and Stubben, 2018; Illmanen, Chandra, and McQuinn, 2020). Similarly, Korteweg (2023) argues that "the issues surrounding staleness of NAVs occupies a fair amount of space in the PE literature, although there is some debate whether it's a bug or a feature" (p. 47) because the resulting lagging and smoothing may, somewhat artificially, appear to reduce the volatility and market exposure of private allocations.

Our conclusion that private funds have provided meaningful diversification benefits to public investors warrants a few comments. On the one hand, it is consistent with two recent studies of diversification benefits also based on PCU data. Goetzmann, Gourier, and Phalippou (2019) find that principal components extracted from unsmoothed quarterly returns are unspanned by public market factors and conclude that "This may help to understand why institutional investors regard private markets as a source of diversification" (p. 3). Brown, Hu, and Kuhn (2021) add randomly selected private funds to an allocation with broad exposure to public equities and bonds and conclude that "[public] investors almost always benefit from diversifying their portfolios with private market exposure" (p. 4). On the other hand, it is important to stress that our results are in aggregate. A given private fund may or may not be expected to provide a public investor with diversification benefits depending on the investor's goals, overall asset allocation, liquidity needs, and time horizon (see Asness, Krail, and Liew 2001, for a similar point on hedge funds). In addition, while we adjust for lagging and smoothing in standard ways, recent literature has suggested more advanced approaches, though these are also substantially more complicated (e.g., Couts, Goncalves, and Rossi 2020; Brown, Ghysels, and Gredil 2023).

The markedly higher asset correlations we find post-2007, i.e., after the adoption of fair value accounting, are indicative of a dramatic impact of mark-to-market valuation. Whether the effect is causal is beyond the scope of this paper, but we note the consistency across the private asset classes we consider, the robustness to excluding the 2008–2010 crisis years, and the similar effects for European private funds documented by Welch and Stubben (2018). A deeper understanding of the implications of fair value accounting, and valuation guidelines more generally, is an interesting avenue for future research, especially if reporting standards and industry guidelines continue to call for more transparent and comparable valuation techniques.³⁰

The increasing interest in (and demand for) private allocations over recent decades may have implications for the returns investors can expect going forward.³¹ "Democratization" of private markets has given a broader set of investors access to the asset class through various vehicles.³² In addition, the private asset management industry has become increasingly institutionalized, with a few large players making up the vast majority of assets under management (referred to as "The Big Four" in Phalippou's (2020) critique of the industry, which features rebuttals from said four managers). Merton (1986) is a classic reference for the equilibrium effects of the size of the investor base on an asset's expected return, with the key implication that "less well-known [assets] with smaller investor bases tend to have relatively larger expected returns" (p. 507).

^{30.} In June 2022, the FASB amended the US accounting standard on fair value measurement (ASC 820) through Accounting Standards Update (ASU) 2022-03, which states that contractual restrictions with respect to equity securities (such as an underwriter's lockup) should be ignored (and a discount may no longer be applied) when estimating fair value. In December 2022, the IPEV guidelines were updated to incorporate ASU 2022-03 and other changes to international accounting standards. On August 23, 2023, the US Securities and Exchange Commission adopted new rules on the regulation of private fund advisers that, among other things, require advisers to (1) provide investors with quarterly statements detailing performance, fees, and expenses; (2) obtain an annual audit; (3) obtain a fairness or valuation opinion on adviser-led secondary transactions; and (4) prohibit certain types of preferential treatment of certain investors.

^{31.} Ernst & Young report in April 2024 that "for more than a decade, private markets have enjoyed a remarkable period of sustained growth, more than doubling from US\$9.7 trillion in assets under management (AUM) in 2012, and are estimated to have reached \$24.4 trillion AUM by the end of 2023." Source: "Are You Harnessing the Growth and Resilience of Private Capital?" EY, April 4, 2024.

^{32.} The Chartered Alternative Investment Analyst (CAIA) Association reported in January 2024 that "there is a multi-pronged industry approach to expanding access of private markets to individuals. Some of the efforts include: Private wealth managers collaborating with private asset managers to develop pooled investment vehicles for the private wealth managers' accredited investor clients, granting them access to funds that might have been out of reach due to minimum investment amounts and administrative burdens [...]." Source: "The Democratization of Private Markets Closing the Information Gap," CAIA Association, January 23, 2024.

Intuitively, information acquisition is costlier for such assets. As a result, investors pay lower prices and require higher returns for investing in them, all else equal. "If a sufficient quantity of such investments were undertaken," Merton concludes, "this 'extra' excess return would disappear," albeit adding that "the time frame over which such corrective action takes place can be considerable and even in the long run, it may not be complete" (p. 507-8). Ascertaining the effects of a larger private investor base is an interesting avenue for future research.

Lastly, we suggest private fund managers and data providers adopt greater transparency. For managers, openness around deal prices, valuation techniques, and the incorporation of mark-to-market provisions would substantially ease investors' monitoring burden. On a similar note, using volatility and beta estimates adjusted for lagging and smoothing should be standard in marketing materials (see Section 4). For data providers, allowing access to more granular (yet still anonymized) data at the manager or fund level is key for research on important topics such as sources of added value and performance persistence. The PCU is arguably more comprehensive and of higher quality than other private fund databases, but the restricted access (i.e., only vintage-level data, subject to a five-fund minimum) severely limits what we are able to study. Alleviating some of these restrictions is key to a deeper understanding of private fund performance.

Appendix A

Size of Investable Universe

 TABLE A1: Estimated Market Value by Asset Class , as of December 31, 2022

	Asset Class	Source/Proxy	Value (\$MM)	Percent of Total Estimated Value
	US Equities	MSCI USA IMI (ex-REITs) ¹	36,601.383	25.8%
	Developed ex-US Equities	MSCI World ex-USA IMI (ex-REITs) ¹	18,316,881	12.9%
Global Public Equities and REITs	Emerging Equities	MSCI Emerging Markets IMI (ex-REITs) 1	7,244,426	5.1%
	Global REITs	REITs portion of MSCI IMI indices listed above	1,648,330	1.2%
Total Global Equities and REITs			63,811,474	45.0%
	USD Investment-Grade Bonds	Bloomberg Global Aggregate Bond Index ¹	26,936,266	19.0%
	USD High-Yield Bonds	Bloomberg Global High Yield Bond Index ¹	1,926,922	1.4%
	Global ex-USD InvGrade Bonds	Bloomberg Global Aggregate Bond Index ^{1,2}	32,617,850	23.0%
Global Public Fixed Income	Global ex-USD High-Yield Bonds	Bloomberg Global High Yield Bond Index ^{1,2}	477,302	0.3%
	US Inflation-Linked Bonds	Bloomberg US TIPS Index	1,203,428	0.8%
	Developed ex-USA InflLinked Bonds	FTSE World Inflation-Linked Securities Index	1,588,467	1.1%
	Municipal Bonds	ICE BofA US Municipal Securities Index	1,058,399	0.7%
otal Fixed Income			65,808,634	46.4%
otal Securities Traded in econdary Markets			129,620,109	91.4%
	Private Equity	Preqin	7,792,600	5.5%
Private and Alternative Assets	Private Real Estate	Preqin	1,587,900	1.1%
	Private Debt	Preqin	1,477,200	1.0%
	Infrastructure	Preqin	1,143,400	0.8%
	Natural Resources	Preqin	228,800	0.2%
otal Securities without Developed Secondary Markets			12,230,900	8.6%
Overall Total			141,851,009	100.0%

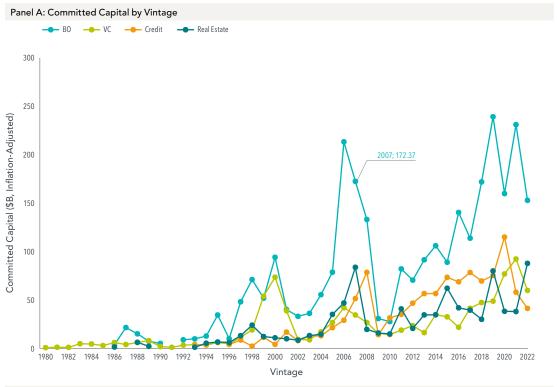
This table shows estimated market values in millions of US dollars and as a percentage of total by asset class. Data are sourced from MSCI, Bloomberg, FTSE, and Preqin. Estimated values for private and alternative assets are sourced from Preqin. Funds of funds and secondaries are excluded in the estimates to avoid double counting. MSCI data © MSCI Inc. 2024; all rights reserved. Bloomberg data provided by Bloomberg. FTSE fixed income indices © FTSE Fixed Income LLC 2024. Preqin data © Preqin Ltd. 2024.

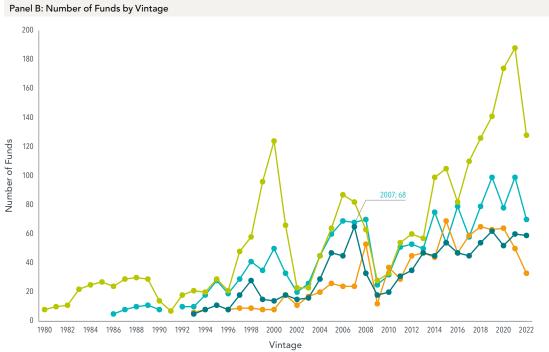
^{1.} Free-float-adjusted values.

^{2.} Ex USD values are computed by subtracting the market value of USD issued bonds in the index from the total index market value.

Appendix B

FIGURE B1: Committed Capital and Number of Funds by Asset Class and Vintage





This figure shows inflation-adjusted committed capital (in billions of 202404 dollars) and the number of firms by asset class and vintage as of December of each vintage year. The highlighted points correspond to the 2007 BO vintage, used as an example in Section 2.3. See Appendix L for disclosures. Data are annual and cover 1980 to 2022. Note that we do not have data for the 1991 BO vintage and the 1995 private credit vintage because the PCU reports only four and three funds in these vintages.

Appendix C

C.1 Effects of Leverage on KS-PME

To see the effects of leverage on KS-PME, consider a one-period deal and assume no fees or carried interest. This is without loss of generality but greatly simplifies the notation.

Suppose the GP can make a total investment of size K_0 at time 0. Given K_0 , let $PIC_0(K_0)$ be the LP's paid-in capital at time 0. Similarly, let $V_1(K_0)$ be the deal's exit value and $DST_1(K_0)$ be the distribution to the LP at time 1.

Consider a time-t cash flow, X_t , for t=0,1. Define $PV(X_t)=\mathbf{E}_0[X_t/(1+r_{0,t}^B)]$ as its present value at time 0 under the Rubenstein CAPM, where $r_{0,t}^B=B_t/B_0-1$ is the benchmark's return between times 0 and t. Since $r_{0,0}^B=0$, we of course have $PIC_0(K_0)=K_0$.

Without leverage, the LP contributes the entire investment amount, so $PIC_0(K_0) = K_0$, and the distribution to the LP is simply the deal's exit value, $DST_1(K_0) = V_1(K_0)$. The present value of the unlevered deal is therefore $PV(V_1(K_0)) - PV(K_0)$ and unlevered KS-PME is

$$KSPME_{U} = \frac{PV(V_{1}(K_{0}))}{PV(K_{0})}.$$

Now suppose the GP can finance part of K_0 with debt. Specifically, write $K_0 = D_0 + C_0$, where D_0 is the borrowed amount, and the remainder, $C_0 = K_0 - D_0 = \operatorname{PIC}_0(K_0)$, is the LP's contribution. Note that the investment size, K_0 , stays fixed but the financing mix changes. Also, suppose the exit value, $V_1(K_0)$, does not depend on the financing mix. This is the relevant case because it implies the GP cannot increase the exit value through debt alone.

Let D_1 be the debt's repayment value at time 1. The distribution to the LP is then given by $DST_1(K_0) = V_1(K_0) - D_1$. Since now $PIC_0(K_0) = C_0$, the PV of the levered deal is

$$PV(V_1(K_0)) - PV(D_1) - PV(C_0) = PV(V_1(K_0)) - PV(D_1) - (PV(K_0) - PV(D_0))$$

and the levered KS-PME is

$$KSPME_{L} = \frac{PV(V_{1}(K_{0})) - PV(D_{1})}{PV(C_{0})} = \frac{PV(V_{1}(K_{0})) - PV(D_{1})}{PV(K_{0}) - PV(D_{0})}.$$

With fair debt pricing, $PV(D_1) = PV(D_0)$, so the PV of the deal is the same with or without leverage. This is again the relevant case because it means the GP cannot add value by simply changing the financing mix with fairly priced debt. Setting $L = PV(D_1) = PV(D_0)$, we can study the effects of leverage on KS-PME through the derivative with respect to L:

$$\frac{\mathrm{d}}{\mathrm{d}L} \text{KSPME}_{L} = \frac{PV(V_{1}(K_{0})) - PV(K_{0})}{(PV(K_{0}) - L)^{2}} = \frac{\text{KSPME}_{U} - 1}{\frac{1}{PV(K_{0})} (PV(K_{0}) - L)^{2}}.$$

When the exit value is independent of the financing mix and debt is fairly priced, KS-PME is (i) decreasing in leverage when unlevered KS-PME is strictly less than 1; (ii) independent of leverage when unlevered KS-PME is equal to 1; and (iii) increasing in leverage when unlevered KS-PME is strictly greater than 1. Put simply, under the above assumptions, leverage pushes unlevered KS-PME further above or below 1 but cannot cause it to cross 1. For a crossing to happen, either the exit value changes with debt or the debt is not fairly priced. In particular, if the GP can increase the exit value through debt and/or has access to "cheap" debt, the PV of the levered fund may be higher than that of the unlevered fund, and KS-PME can cross from below to above 1.

C.2 Effects of Risky Debt on KS-PME

For simplicity, the numerical example in **Section 2.6** considered risk-free debt and fair debt pricing. Here, we maintain the example's setup but consider the effects of risky debt and deviations from fair debt pricing.

Recall the setup (see additional details in **Section 2.6**). In a one-period binomial tree with equally likely states, the benchmark return is either 40% or -20%. Under the Rubenstein CAPM, the corresponding risk-free rate is 1.81%. The GP can invest \$1,000 in a deal with a constant excess return, α , above the benchmark. In the absence of leverage, the PV of the deal is $1/1.4 + \alpha = 0.8 + \alpha$)

$$1000 \times \frac{1}{2} \left(\frac{1.4 + \alpha}{1.4} + \frac{0.8 + \alpha}{0.8} \right) - 1000 \text{ and KS-PME is } \frac{1}{2} \left(\frac{1.4 + \alpha}{1.4} + \frac{0.8 + \alpha}{0.8} \right).$$

Now suppose the GP finances \$800 with debt (this is higher than the \$400 in **Section 2.6** to allow for risky debt when α is nonnegative). The LP contributes the remaining \$200. The debt's promised repayment is 800(1 + y), where y is the yield, but it is risky because the actual payment to creditors depends on whether the deal's exit value can cover the obligation. Specifically, define the debt's state-dependent repayment values as

$$D_1^+(y,\alpha) = \min\{800(1+y), 1000(1.4+\alpha)\},\$$

$$D_1^-(y,\alpha) = \min\{800(1+y), 1000(0.8+\alpha)\}.$$

Let
$$PV[D_1(y,\alpha)] = \frac{1}{2} \left(\frac{D_1^+(y,\alpha)}{1.4} + \frac{D_1^-(y,\alpha)}{0.8} \right)$$
 be the debt's PV. The levered deal's PV becomes

$$\frac{1}{2} \left(\frac{1000(1.4 + \alpha) - D_1^+(y, \alpha)}{1.4} + \frac{1000(0.8 + \alpha) - D_1^-(y, \alpha)}{0.8} \right) - 200$$

$$= \left[1000 \times \frac{1}{2} \left(\frac{1.4 + \alpha}{1.4} + \frac{0.8 + \alpha}{0.8}\right) - PV[D_1(y, \alpha)]\right] - [1000 - 800],$$

and the corresponding levered KS-PME is given by

$$\frac{1000 \times \frac{1}{2} \left(\frac{1.4 + \alpha}{1.4} + \frac{0.8 + \alpha}{0.8} \right) - PV[D_1(y, \alpha)]}{1000 - 800}.$$

With fair debt pricing, the creditors break even, so the corresponding yield, y_{Fair} , solves $PV[D_1(y_{Fair},\alpha)]=800$ for a given α . In that case, the deal's PV is the same with or without leverage, and levered KS-PME moves further away from 1 compared to unlevered KS-PME. If, however, the debt is "cheap" from the GP's perspective in the sense that $y < y_Fair$, then leverage can increase the deal's PV and cause KS-PME to cross from below 1 to above 1. In the following, we demonstrate this using comparative statics with respect to α and y.

When $\alpha = -10\%$, the break-even yield is $y_{Fair} = 26.9\%$, for a promised repayment of \$1,015. At this yield, the creditors receive the full amount in the "good" state (when the benchmark return is 40%), but the GP defaults in the "bad" state (when the benchmark return is -20%) and creditors recover \$700. Leverages shrinks KS-PME from 0.90 to 0.51.

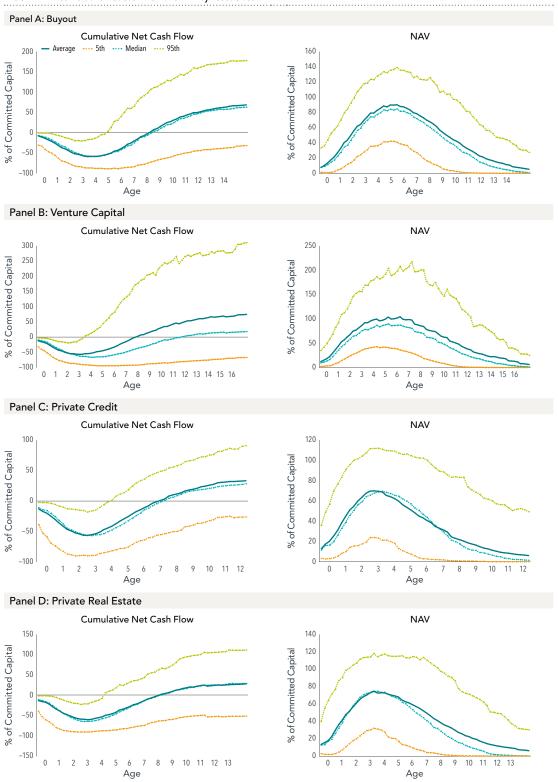
If we maintain $\alpha = -10\%$ but assume "cheap" debt with a yield of y=10%, levered KS-PME is 0.75. In fact, the yield would have to be 0% for levered KS-PME to equal unlevered KS-PME (0.90), and it would have to be -7.5% for levered KS-PME to increase to 1.

When α = 0%, we have y_{Fair} = 5.0% and KS-PME equals 1 with or without leverage. If instead y = 2.5%, levered KS-PME is 1.04. If y = $r_{\scriptscriptstyle F}$ = 1.81%, levered KS-PME is 1.05.

Lastly, when α = 10%, we have $y_{\text{Fair}} \approx r_{F'}$ and leverage increases KS-PME from 1.10 to 1.49. The yield would have to be y = 17.5% for levered KS-PME to drop to 1.

Appendix D

FIGURE D1: Cumulative Net Cash Flow and NAV by Asset Class

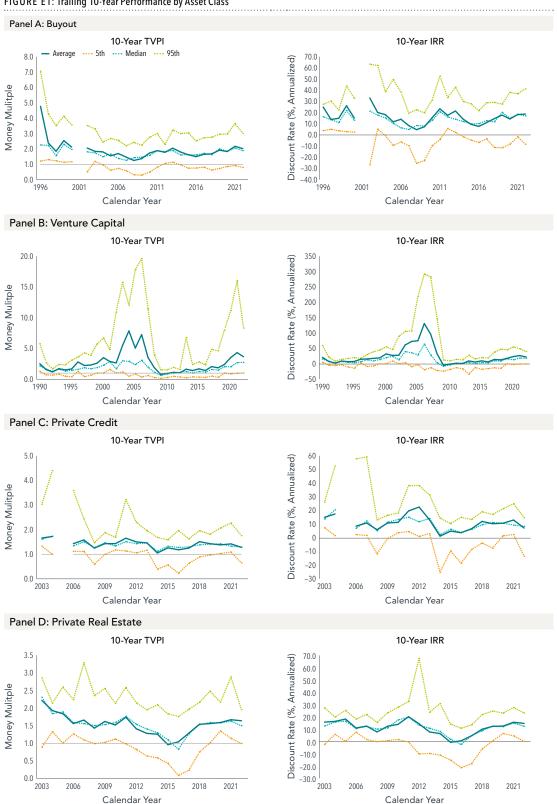


Past performance is not a guarantee of future results.

In USD. This figure shows weighted averages of selected statistics from the sample distributions of cumulative net cash flow and NAV as a function of vintage age. The averaging is across vintages with weights determined by inflation-adjusted total committed capital (using US CPI and expressed in 2022Q4 dollars). Age is measured in years relative to the end of Q4 of the vintage year. All cash flows and NAVs are net of management fees and carried interest. See **Appendix L** for disclosures. The sample is all North American private funds covered by the PCU. Data are quarterly, end with 2022, and start in 1980 for venture capital, 1986 for buyouts, and 1993 for private credit and private real estate.

Appendix E

FIGURE E1: Trailing 10-Year Performance by Asset Class



Past performance is not a guarantee of future results.

In USD. Note that we do not have data for the 1991 BO vintage and the 1995 private credit vintage because the PCU reports only four and three funds in these vintages, respectively. See Appendix L for disclosures.

Appendix F

TABLE F1: Statistical Tests for Relative Performance Measures

•••••••••••••••••••••••••••••••••••••••	Equal-Weighted Ave	erages across Vintages	Capital-Weighted Averages across Vintages			
Benchmark	KS-PME	DA	KS-PME	DA		
	(t-test vs. 1)	(t-test vs. 0)	(t-test vs. 1)	(t-test vs. 0)		
Panel A: Buyout (Vintages Aged 10	0–15 Years; <i>N</i> = 26)					
S&P 500 Index	1.21	4.52	1.17	3.68		
	(4.38)	(3.79)	(2.51)	(2.45)		
Large Value	1.15	3.07	1.15	3.05		
	(2.46)	(2.45)	(2.66)	(2.67)		
Large High Profitability	1.14	3.24	1.08	2.02		
	(3.42)	(3.09)	(1.27)	(1.26)		
Small Value	1.06	0.95	1.01	0.21		
	(0.86)	(0.68)	(0.23)	(0.15)		
Small Growth	1.15	3.01	1.05	1.18		
	(2.62)	(3.10)	(0.96)	(1.01)		
Panel B: Venture Capital (Vintages	Aged 10–17 Years; N	= 33)				
S&P 500 Index	1.43	8.80	1.19	4.15		
	(1.45)	(1.43)	(1.30)	(1.63)		
Large Value	1.46	9.19	1.21	4.31		
	(1.41)	(1.34)	(1.13)	(1.08)		
Large High Profitability	1.36	7.82	1.09	2.62		
	(1.21)	(1.23)	(0.65)	(0.99)		
Small Value	1.45	9.05	1.11	2.25		
	(1.29)	(1.25)	(0.51)	(0.48)		
Small Growth	1.51	10.03	1.13	2.75		
	(1.47)	(1.42)	(0.62)	(0.69)		
Panel C: Private Credit (Vintages A	Aged 10–12 Years; <i>N</i> =	= 19)				
Bloomberg US Credit Index	1.13	3.88	1.11	3.33		
	(3.01)	(3.04)	(3.68)	(3.58)		
Bloomberg US High Yield Index	1.06	1.76	1.00	0.28		
	(1.01)	(1.27)	(0.06)	(0.31)		
Panel D: Private Real Estate (Vinta	ges Aged 10–14 Year	s; N = 20)				
Dow Jones US Select REIT Index	0.96	-0.92	0.89	-2.12		
	(-0.66)	(-0.68)	(-1.39)	(-1.26)		
Fama/French Real Estate Industry Portfolio	1.15	3.29	1.04	1.16		
	(1.53)	(1.68)	(0.41)	(0.60)		

Past performance is not a guarantee of future results.

In USD. This table shows equal-weighted and capital-weighted averages across vintages of the average fund's Kaplan-Schoar public market equivalent (KS-PME) and the Gredil-Griffith-Stucke direct alpha (DA) along with the corresponding \mathbf{t} -statistics (against a value of 1 for KS-PME and a value of 0 for DA). DA is in annualized %. Capital-weighted averages and corresponding \mathbf{t} -statistics are computed using weighted least squares, where capitalization values are inflation-adjusted and in 2022O4 dollars. All \mathbf{t} -statistics use Newey-West standard errors with automatic lag selection, and estimates with $|\mathbf{t}| \ge 2$ are indicated in bold. We only consider vintages with a minimum age of 10 years as of 2022O4, measured since O4 of the vintage year. Buyouts: vintages aged 0-15 years, i.e., the 1986–2012 vintages excluding the 1995 vintage for which we do not have data due to the PCU only reporting four funds. Venture capital: vintages aged 10-17 years, i.e., the 1993–2012 vintages excluding the 1995 vintage sexcluding the 1995 vintage for which we do not have data due to the PCU only reporting three funds. Private real estate: vintages aged 10-14 years, i.e., the 1993–2012 vintages. See **Appendix L** for disclosures.

Appendix G

Overview of Literature on BO and VC Factor Exposures

G.1 BO factor exposures

Kaplan and Schoar (2005) regress BO funds' lifetime IRRs on the average annual return to the S&P 500 Index in the five years after a fund is raised and find a coefficient of 0.4 (with a standard error of 0.3).

Franzoni, Nowak, and Phalippou (2012) use the gross-of-fees cash flows of buyout investments to estimate a version of Cochrane's (2005) log-normal linear factor model and find a beta of 0.9–1.4, a negative but unreliable size loading, and a reliably positive value loading.

Driessen, Lin, and Phalippou (2012) specify BO funds' lifetime IRRs as a linear function of exposures to realized factor returns and find a beta of 1.3–1.7, a negative but unreliable size loading, and a positive but unreliable value loading.

Ang, Chen, Goetzmann, and Phalippou (2018) jointly estimate the time series of BO fund returns and factor exposures and find a beta of 1.3–1.4, a negative but unreliable size loading, and a positive value loading whose reliability depends on the proxy for the market return.

Cao and Lerner (2009) use time-series regressions of the returns to portfolios of reverse BOs (i.e., initial public offerings of firms previously bought out by private equity investors) and find a beta of 1.2–1.3, a reliably positive size loading, and a small and unreliable value loading.

Jegadeesh, Kraussl, and Pollet (2015) use the returns to publicly listed funds-of-private equity funds and estimate that the buyout-focused funds have a beta of 0.7–1.0, a reliably positive size loading, and a positive but unreliable value loading.

Welch and Stubben (2018) find, in panel regressions, that "Following the change to fair value accounting, the average market beta based on reported NAVs of European buyout funds more than tripled from 0.26 to 0.92." (p. 3).

Brown, Ghysels, and Gredil (2023) use fund-level PCU data and a state space model to "nowcast" weekly NAVs and returns and estimate a beta around 1 and reliably positive size and value loadings for BOs.

Korteweg and Westerfield (2022) argue that "The value loading is indicative of the traditional buyout strategy of taking over struggling firms and turning them around. With the recent expansion of buyout into other strategies, including more early stage investments, these loadings may be weaker going forward" (p. 34).

G.2 VC factor exposures

Kaplan and Schoar (2005) find that VC funds' lifetime IRRs have a beta of 1.2 when regressed on the average annual return to the S&P 500 Index in the five years after a fund is raised.

Cochrane (2005) estimates a log-normal linear factor model using data on VC funding rounds and finds a beta of 1.9 when using the total return from the first financing round to an initial public offering or an acquisition, but a beta of 0.6 when using round-to-round returns.

Korteweg and Sørensen (2010) estimate an extension of Cochrane's (2005) model using round-to-round returns and find a beta of 2.8, a reliably positive size loading, and a reliably negative value loading.

Driessen, Lin, and Phalippou (2012) find that VC funds have a beta of 2.4–2.7, a positive but unreliable size loading, and a negative but unreliable value loading.

Ang, Chen, Goetzmann, and Phalippou (2018) jointly estimate the time series of VC fund returns and factor exposures and find a beta of 1.1–1.6, a positive but unreliable size loading, and a negative but at most marginally significant value loading.

Jegadeesh, Kraussl, and Pollet (2015) use the returns to publicly listed funds-of-private equity funds and estimate that the venture-focused funds have a beta of 0.9–1.0, a positive and marginally significant size loading, and a positive but unreliable value loading.

Brown, Ghysels, and Gredil (2023) use fund-level PCU data and a state space model to "nowcast" weekly NAVs and returns and estimate that VCs have a beta around 1.7, a reliably positive size loading, and a reliable negative value loading.

Appendix H

Five-Factor Regressions for Private Real Estate

TABLE H1: Factor Regressions for Private Real Estate Using the Fama/French Five-Factor Model

Panel A: Summary Statistics

	199404-200704					
	Raw	Unsmoothed	Fama/French US Market			
Mean Excess Return (%/Quarter)	3.28	3.28	2.00			
Volatility (%, ann.)	6.60	12.12	16.81			
Volatility (%, Newey-West)	7.44	12.36	16.75			
t-Statistic	7.23	3.93	1.73			
t-Statistic (Newey-West)	6.41	3.86	1.73			

2008Q1-2022Q4							
Raw	Unsmoothed	Fama/French US Market					
0.94	0.90	2.41					
9.70	18.59	18.47					
17.36	17.02	18.73					
1.49	0.75	2.02					
0.84	0.82	1.99					

Panel B: Time-Series Regressions of Quarterly Net Excess Returns (Newey-West t-statistic in parentheses; reliable estimates in bold)

1994Q4-2007Q4

100404 200704

200801	-202204

	(1) Raw Lag 0	(2) Raw Lags 0–1	(3) Raw Lags 0-1	(4) Unsmoothed Lags 0-1	(5) Raw Lag 0	(6) Raw Lags 0-5	(7) Raw Lags 0-5	(8) Unsmoothed Lags 0–5
Intercept (%/Quarter)	3.06 (6.74)	2.88 (6.81)	2.79 (6.04)	2.23 (2.42)	0.54 (0.45)	-1.90 (-3.20)	-1.78 (-2.67)	-1.87 (-2.59)
$R_M - R_F$	0.11 (3.33)	0.19 (3.57)	0.23 (2.91)	0.37 (2.26)	0.17 (2.11)	1.10 (8.15)	1.12 (10.24)	0.99 (5.44)
SMB			-0.11 (-1.10)	-0.03 (-0.15)			-0.43 (-2.25)	-0.03 (-0.08)
HML			0.33 (2.46)	0.67 (2.63)			0.05 (0.22)	-0.24 (-0.61)
RMW			-0.08 (-0.60)	-0.24 (-1.10)			-0.08 (-0.26)	0.09 (0.31)
CMA			-0.23 (-1.17)	-0.25 (-0.60)			0.14 (0.40)	0.32 (0.49)
Adjusted R ² (%)	6.1	8.0	5.7	6.2	8.44	69.07	65.49	31.09
CI (Adjusted R ² , 95%)	(-1.5, 21.2)	(-2.2, 18.0)	(-13.5, 13.6)	(-13.1, 14.3)	(-1.7, 39.7)	(42.1, 75.6)	(48.6, 74.5)	(-10.5, 47.1)
N (Quarters)	53	53	53	53	60	60	60	60
Degrees of Freedom	51	50	42	42	 58	53	29	29

Past performance is not a guarantee of future results.

In USD. Panel A shows summary statistics for aggregate quarterly net excess private real estate returns and quarterly Fama/French market excess returns before the adoption of fair value accounting (1994Q4-2007Q4) and after (2008Q1-2022Q4). Unsmoothed returns are based on an AR(6) before the adoption of fair value accounting and an AR(3) post-2007, determined using the partial autocorrelation function. Panel B shows time-series regressions on the quarterly returns to the Fama and French (2015) factors with Newey-West t-statistics for the null of zero in brackets, and estimates with |t| ≥ 2 are indicated in bold. Quarterly factor returns are the differences between the compound returns to the long and short sides. The regressions correct for lagging by summing the coefficients on lagged regressors, where the number of lags is chosen based on the significance of the coefficients on lagged market excess returns. Test-statistics for the summed coefficients are based on the estimated Newey-West variance covariance matrix. All Newey-West adjustments use automatic lag selection. Each 95% confidence interval for adjusted R² uses the bias-corrected and accelerated method on 10,000 nonparametric bootstrap samples. See Appendix L for disclosures.

Appendix I

TABLE 11: Factor Regressions over the 2011-2022 Period

Panel A: Summary Statistics for Unsmoothed Aggregate Net Excess Returns, 2011Q1–2022Q4								
BO VC Credit Real Estate								
Mean Excess Return (%/Quarter)	3.16	3.61	1.80	2.23				
t-Statistic (Newey-West)	4.61	2.38	3.80	4.96				

Panel B: Regressions of Unsmoothed Aggregate Net Excess Returns, 2011Q1–2022Q4 (Newey-West t-statistic in parentheses; reliable estimates in bold)

(Newey-west t-statistic in parentneses, reliable estimates in bold)							
	BO (1) Lags 0-3	VC (2) Lags 0-2	Credit (3) Lags 0-3	Real Estate (4) Lags 0-4			
Intercept (%/Quarter)	1.33 (5.27)	-0.35 (-0.41)	0.78 (2.24)	1.04 (2.51)			
$R_M - R_F$	0.53 (5.90)	0.99 (4.36)					
SMB	0.35 (3.42)	0.69 (2.05)					
HML	0.05 (0.99)	-0.77 (-2.74)					
RMW	0.25 (2.52)	1.25 (2.65)					
CMA	-0.06 (-1.04)	0.02 (0.06)					
$R_{HY} - R_F$			0.84 (5.77)				
$R_{HY; Long} - R_{HY; Intem.}$			-0.05 (-0.34)				
$R_{REIT} - R_F$				0.50 (5.33)			
Adjusted R ² (%)	90.4	69.9	70.3	36.4			
CI (Adjusted R ² , 95%)	(84.1, 93.5)	(48.5, 74.1)	(44.9, 84.0)	(6.3, 60.0)			
N (Quarters)	48	48	48	48			
Degrees of Freedom	27	32	39	42			

Past performance is not a guarantee of future results.

In USD. Panel A shows summary statistics for unsmoothed aggregate quarterly net excess returns after the 2008–2010 crisis years (201101–202204). Unsmoothed returns are based on an AR(1) for BOs and private credit and an AR(3) for VCs and private real estate, determined using the partial autocorrelation function. Panel B shows time-series regressions on the quarterly returns to the Fama and French (2015) factors with Newey-West t-statistics for the null of zero in brackets, and estimates with Jt | 2 2 are indicated in bold. Quarterly factor returns are the differences between the compound returns to the long and short sides. The regressions correct for lagging by summing the coefficients on lagged regressors, where the number of lags is chosen based on the significance of the coefficients on lagged market excess returns. Test-statistics for the summed coefficients are based on the estimated Newey-West variance-covariance matrix. All Newey-West adjustments use automatic lag selection. Each 95% confidence interval for adjusted R² uses the bias-corrected and accelerated method on 10,000 nonparametric bootstrap samples. See Appendix L for disclosures.

Appendix J

TABLE J1: Listed Private Equity Management Companies

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept (%/Quarter)	2.57 (1.60)	-0.81 (-0.59)	1.04 (0.76)	-1.18 (-1.18)	1.42 (1.18)	-1.46 (-1.78)	-0.85 (-1.05)	0.38 (0.37)
$\overline{\rm IRR}_{BO}^{Unsm} - R_F$		1.13 (2.31)				0.30 (1.07)	0.14 (1.63)	0.10 (1.11)
$\overline{\rm IRR}_{VC}^{Unsm} - R_F$			0.54 (7.26)			0.24 (3.18)	0.12 (1.40)	0.16 (1.99)
$\overline{\rm IRR}_{Credit}^{Unsm} - R_F$				1.93 (8.25)		1.40 (4.56)	0.70 (2.49)	0.47 (1.65)
$\overline{\text{IRR}}_{Real\ Estate}^{Unsm} - R_F$					0.66 (3.21)	-0.15 (-0.80)	-0.16 (-0.97)	-0.14 (-0.88)
R_M - R_F							0.90 (6.21)	0.81 (6.74)
SMB								0.09 (0.34)
HML								0.69 (3.13)
RMW								-0.58 (-2.47)
CMA								-0.44 (-1.53)
Adjusted R ² (%)		28.8	32.8	48.7	7.9	54.5	65.4	70.3

Past performance is not a guarantee of future results.

In USD. This table shows time-series regressions of the quarterly excess returns to the LPX America Listed Private Equity Index (Total Return) on the excess unsmoothed aggregate quarterly net returns to private funds by asset class. See Appendix A10 for a definition of the LPX index. The unsmoothed return series are plotted in Exhibit 5. Newey-West t-statistics for the null of zero are shown in brackets, and estimates with $|t| \ge 2$ are indicated in bold. All Newey-West adjustments use automatic lag selection. Specifications 7 and 8 control for the quarterly returns to the Fama and French (2015) factors. Quarterly factor returns are the difference s between the compound returns to the long and short sides. Data are quarterly and cover 199803 through 202204, where the start date is determined by the availability of the LPX index. See Appendix L for disclosures.

Appendix K

Index Definitions

Dimensional US Large Cap Value Index

January 1975-present: Compiled by Dimensional from CRSP and Compustat data. The index composition consists of large cap companies in the eligible market whose relative price is in the bottom 30% of the large cap market after the exclusion of utilities, companies lacking financial data, and companies with negative relative price. The index emphasizes securities with higher profitability, lower relative price, and lower market capitalization. Profitability is defined as operating income before depreciation and amortization minus interest expense divided by book equity. The eligible market is composed of securities of US companies traded on the NYSE, NYSE MKT (formerly AMEX), and Nasdaq Global Market. Exclusions: non-US companies, REITs, UITs, and investment companies. The index has been retrospectively calculated by Dimensional and did not exist prior to March 2007. Accordingly, the results shown during the periods prior to March 2007 do not represent actual returns of the index. Other periods selected may have different results, including losses. The calculation methodology for the index was amended in January 2014 to include profitability as a factor in selecting securities for inclusion in the index.

Prior to January 1975: Compiled by Dimensional from CRSP and Compustat data. The index composition consists of large cap companies in the eligible market whose relative price is in the bottom 25% of the US Large Cap Index after the exclusion of utilities, companies lacking financial data, and companies with negative relative price. The eligible market is composed of securities of US companies traded on the NYSE, NYSE MKT (formerly AMEX), and Nasdag Global Market. Exclusions: non-US companies, REITs, UITs, and investment companies.

Dimensional US Large Cap High Profitability Index

Compiled by Dimensional from CRSP and Compustat data. Consists of companies with market capitalizations above the 1,000th largest company whose profitability is in the top 35% of all large cap companies after the exclusion of utilities, companies lacking financial data, and companies with negative relative price. The index emphasizes companies with lower relative price, higher profitability, and lower market capitalization. Profitability is defined as operating income before depreciation and amortization minus interest expense divided by book equity. The eligible market is composed of securities of US companies traded on the NYSE, NYSE MKT (formerly AMEX), and Nasdaq Global Market. Exclusions: non-US companies, REITs, UITs, and investment companies. The index has been retroactively calculated by Dimensional and did not exist prior to December 2016. Accordingly, the results shown during the periods prior to December 2016 do not represent actual returns of the index. Other periods selected may have different results, including losses.

Dimensional US Small Cap Value Index

January 1975–present: Compiled by Dimensional from CRSP and Compustat data. The index composition is a subset of the US Small Cap Index. The subset is defined as companies whose relative price is in the bottom 35% of the US Small Cap Index after the exclusion of utilities, companies lacking financial data, and companies with negative relative price. The eligible market is composed of securities of US companies traded on the NYSE, NYSE MKT (formerly AMEX), and Nasdaq Global Market. Exclusions: non-US companies, REITs, UITs, investment companies, and companies with the lowest profitability within the small cap value universe. The index also excludes those companies with the highest asset growth within the small cap universe. Profitability is defined as operating income before depreciation and amortization minus interest expense divided by book equity. Asset growth is defined as change in total assets from the prior fiscal year to current fiscal year. The index has been retrospectively calculated by Dimensional and did not exist prior to March 2007. Accordingly, the results shown during the periods prior to March 2007 do not represent actual returns of the index. Other periods selected may have different results, including losses. The calculation methodology for the index was amended in January 2014 to include profitability as a factor in selecting securities for inclusion in the index. The calculation methodology for the index was amended in December 2019 to include asset growth as a factor in selecting securities for inclusion in the index.

Prior to January 1975: Compiled by Dimensional from CRSP and Compustat data. The index composition is a subset of the US Small Cap Index. The subset is defined as companies whose relative price is in the bottom 25% of the US Small Cap Index after the exclusion of utilities, companies lacking financial data, and companies with negative relative price. The eligible market is composed of securities of US companies traded on the NYSE, NYSE MKT (formerly AMEX), and Nasdaq Global Market. Exclusions: non-US companies, REITs, UITs, and investment companies.

Dimensional US Small Cap Growth Index

Compiled by Dimensional from CRSP and Compustat data. Consists of companies with market capitalizations in the lowest 8% of the total market capitalization of the eligible market whose relative price is in the top 50% of all small cap companies after the exclusion of utilities, companies lacking financial data, and companies with negative relative price. The index excludes companies with the lowest profitability within the small cap growth universe. The index also excludes those companies with the highest asset growth within the small cap universe. Profitability is defined as operating income before depreciation and amortization minus interest expense divided by book equity. Asset growth is defined as change in total assets from the prior fiscal year to current fiscal year. The eligible market is composed of securities of US companies traded on the NYSE, NYSE MKT (formerly AMEX), and Nasdaq Global Market. Exclusions: non-US companies, REITs, UITs, and investment companies. The index has been retrospectively calculated by Dimensional and did not exist prior to December 2012. Accordingly, the results shown during the periods prior to December 2012 do not represent actual returns of the index. Other periods selected may have different results, including losses. The calculation methodology for the index was amended in December 2019 to include asset growth as a factor in selecting securities for inclusion in the index.

Bloomberg US Credit Index

The Bloomberg US Credit Index measures the investment grade, US dollar-denominated, fixed-rate, taxable corporate and government-related bond markets. It is composed of the US Corporate Index and a non-corporate component that includes non-US agencies, sovereigns, supranationals and local authorities. The US Credit Index was called the US Corporate Index until July 2000, when it was renamed to reflect its inclusion of both corporate and non-corporate issuers. The US Credit Index is a subset of the US Government/Credit Index and US Aggregate Index. Index history is available back to 1973. Bloomberg data provided by Bloomberg Finance L.P.

Bloomberg US Corporate High Yield Indices

The Bloomberg US Corporate High Yield Bond Index measures the USD-denominated, high yield, fixed-rate corporate bond market. Securities are classified as high yield if the middle rating of Moody's, Fitch and S&P is Ba1/BB+/BB+ or below. Bonds from issuers with an emerging markets country of risk, based on the indices' EM country definition, are excluded. The US Corporate High Yield Index is a component of the US Universal and Global High Yield Indices. The index was created in 1998, with history backfilled to July 1, 1983. The "Intermediate" subindex corresponds to the subset of constituents with maturities of 1-10 years. The "Long" subindex corresponds to the subset of constituents with maturities of 10-30 years. Bloomberg data provided by Bloomberg Finance L.P.

Morningstar LSTA Leveraged Loan Index

The Morningstar LSTA US Leveraged Loan Index is a market-value weighted index designed to measure the performance of the US leveraged loan market. The starting universe consists of syndicated term leveraged loans that are held within top-tier institutional investor loan portfolios tracked by PitchBook and LCD. Seniority: senior secured. Currency: USD denominated. Minimum initial term: 1 year. Minimum initial spread: Base rate + 125 bps. Minimum initial issue size: \$50 million. Domicile: all loans must be syndicated in the US, but issuers may be of any origin. Note: A loan in default will remain in the index (but does not accrue interest and is excluded from yield calculations) unless it fails to meet the stated eligibility criteria or has been repaid or restructured. ©2024 Morningstar, Inc. All rights reserved.

Dow Jones US Select REIT Index

Total Return in USD. **January 1987-present**: Dow Jones US Select REIT Index; source: Dow Jones Indexes. **January 1978-January 1987**: Dow Jones Wilshire REIT Index; source: Dow Jones Wilshire. Composition: US publicly traded real estate investment trusts weighted by float-adjusted market capitalization. © 2024 S&P Dow Jones Indices LLC, a division of S&P Global. All rights reserved.

Fama and French (2015) factors

The Market, Size, Value, Profitability, and Investment research factors based on 2x3 sorts for the US. Constructed using the 6 value-weight portfolios formed on size and book-to-market, the 6 value-weight portfolios formed on size and operating profitability, and the 6 value-weight portfolios formed on size and investment. Available from Ken French's website: https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

Fama/French US Real Estate Industry Research Portfolio

Based on the Fama/French 49 US Industry Research Portfolios. Each NYSE, AMEX, and NASDAQ stock is assigned to an industry research portfolio at the end of June of year t based on its four-digit SIC code at that time. When possible, this is based on Compustat SIC codes for the fiscal year ending in calendar year t-1. Whenever Compustat SIC codes are not available, it is instead based on CRSP SIC codes for June of year t. Monthly value-weighted returns are computed from July of t to June of t+1. The Real Estate industry corresponds to SIC codes 6500, 6510, 6512-6515, 6517-6519, 6520-6532, 6540-6541, 6550-6553, 6590-6599, 6610-6611.

Available from Ken French's website: https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

NCREIF Property Index

The National Council of Real Estate Investment Fiduciaries (NCREIF) Property Index (NPI) is a quarterly, unleveraged composite total return for private commercial real estate properties held for investment purposes only. All properties in the NPI have been acquired, at least in part, on behalf of tax-exempt institutional investors and held in a fiduciary environment. Only operating apartment, hotel, industrial, office and retail properties are included in the NPI. An operating property is defined as existing and at least 60% leased. The property can be wholly owned or held in a joint venture structure. Although NPI returns are reported on a unlevered basis, there are properties in the NPI that utilize leverage. Each property's return is weighted by its market value. The NPI goes back to Fourth Quarter 1977. Copyright © 2024 National Council of Real Estate Investment Fiduciaries.

NFI-ODCE Net Return Index

The NFI-ODCE, short for National Council of Real Estate Investment Fiduciaries (NCREIF) Fund Index - Open End Diversified Core Equity, is an index of investment returns of 38 open-end commingled funds pursuing a core investment strategy, some of which have performance histories dating back to the 1970s. The NFI-ODCE Net Return Index is capitalization-weighted and reported net of fees. Measurement is time weighted. Open-end funds are generally defined as infinite-life vehicles consisting of multiple investors who have the ability to enter or exit the fund on a periodic basis, subject to contribution and/or redemption requests, thereby providing a degree of potential investment liquidity. The term Diversified Core Equity style typically reflects lower risk investment strategies utilizing low leverage and generally represented by equity ownership positions in stable US operating properties diversified across regions and property types. The inception date is December 31, 1977. Copyright © 2024 National Council of Real Estate Investment Fiduciaries.

LPX America Listed Private Equity Index (Total Return, Net)

Represents the performance of listed private equity fund management companies, which are listed on a North American stock exchange. The index comprises the 30 most highly capitalized and liquid companies and is diversified across private equity investment styles, financing styles, and vintages. Copyright © 2024 LPX AG.

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Appendix L

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