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Pursuing Higher Expected Returns with Duration Constraints

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INTRODUCTION

There are many ways to structure a fixed income portfolio even when a desired duration is required (e.g., matching the duration of a benchmark or a liability). One can invest in bonds with different maturities across the yield curve and hold them until maturity; then, as bonds mature, the proceeds are invested in new bonds so that the average duration is kept at the desired level. Alternatively, one can hold bonds within certain maturity segments and rebalance to maintain the desired duration. While these approaches may be managed to meet the same duration constraints, their expected returns can be quite different because of the current shape and the expected movement (if any) of the yield curve.

In this article, we present a theoretical framework for how to pursue higher expected returns of a fixed income portfolio subject to duration constraints. That framework is then tested using a historical time series of US Treasuries. The results suggest that, by using the information in current yield curves, we can improve expected return in a systematic and reliable way, while still maintaining the desired duration.

THEORETICAL FRAMEWORK

The building block of this analysis is the expected return on a single zero coupon bond with duration n over a holding period of Δt, denoted as $E_0\left(R_{\Delta t}^{(n)}\right)$. The expected return of a portfolio is then the weighted average of the expected returns of the eligible bonds, with durations between zero and a maximum duration, selected for the portfolio: $\sum w_n E_0\left(R_{\Delta t}^{(n)}\right)$, where w_n is the weight of the bond with duration n. The goal is to maximize this expected return while maintaining the duration of the portfolio at D, which can be formalized as the following constrained optimization problem:

$$\begin{array}{ll} \textit{Maximize} \sum w_n E_0\left(R_{\Delta t}^{(n)}\right) \text{ for all } w_n,\ 0 \leq n \leq \max \textit{Duration} \\ \textit{Subject to:} & \sum w_n \cdot n = D \\ & \sum w_n = 1 \\ & 0 \leq w_n \leq 1 \text{ for all } w_n \end{array}$$

The question then becomes, can we say something about the expected return $E_0\left(R_{\Delta t}^{(n)}\right)$ using information currently available at time zero?

Define $y_t^{(n)}$ as the yield-to-maturity at time t for a zero coupon bond with duration n. The current price of the zero coupon bond with duration n and yield to maturity $y_0^{(n)}$ is $P_0^{(n)} = e^{-n\, y_0^{(n)}}$. Over a period of time Δt, the bond duration will become $n - \Delta t$ and its new yield to maturity $y_{\Delta t}^{(n-\Delta t)}$. Its price will be $P_{\Delta t}^{(n-\Delta t)} = e^{-(n-\Delta t)\, y_{\Delta t}^{(n-\Delta t)}}$. The expected return can be written as

$$E_0\left(R_{\Delta t}^{(n)}\right) = E_0\left(\frac{P_{\Delta t}^{(n-\Delta t)}}{P_0^{(n)}} - 1\right) = E_0\left(\frac{e^{-(n-\Delta t)\, y_{\Delta t}^{(n-\Delta t)}}}{e^{-n\, y_0^{(n)}}} - 1\right)$$

To gain more intuition we can decompose the expected return into three parts by multiplying and dividing it by $e^{-(n-\Delta t) y_0^{(n)}}$ and $e^{-(n-\Delta t) y_0^{(n-\Delta t)}}$.

$$E_0\left(R_{\Delta t}^{(n)}\right) = E_0\left(\frac{e^{-(n-\Delta t) y_0^{(n)}}}{e^{-n\, y_0^{(n)}}} \cdot \frac{e^{-(n-\Delta t) y_0^{(n-\Delta t)}}}{e^{-(n-\Delta t) y_0^{(n)}}} \cdot \frac{e^{-(n-\Delta t) y_{\Delta t}^{(n-\Delta t)}}}{e^{-(n-\Delta t) y_0^{(n-\Delta t)}}} - 1\right)$$

$$= \underbrace{e^{\Delta t\, y_0^{(n)}}}_{(1)} \underbrace{e^{-(n-\Delta t)\left(y_0^{(n-\Delta t)} - y_0^{(n)}\right)}}_{(2)} E_0\left(\underbrace{e^{-(n-\Delta t)\left(y_{\Delta t}^{(n-\Delta t)} - y_0^{(n-\Delta t)}\right)}}_{(3)}\right) - 1$$

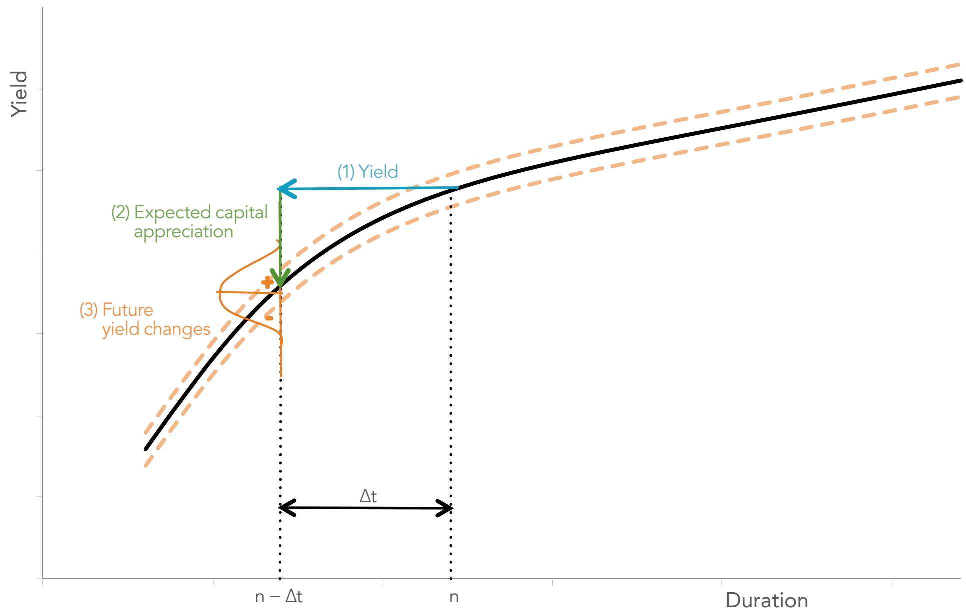
(1) A component related to the bond's current yield $y_0^{(n)}$.

(2) A component related to the bond's expected capital appreciation or depreciation over the next Δt based on the current (at time zero) term structure. Over the next Δt, an n-year bond yielding $y_0^{(n)}$ will have $n - \Delta t$ to maturity with yield $y_0^{(n-\Delta t)}$ due to movements *along* the current yield curve.

(3) A component related to changes *in* the yield curve itself in the next Δt.

Exhibit 1 shows a graphical representation of this decomposition.

Exhibit 1: A Decomposition of Expected Return



For illustrative purposes only.

Terms (1) and (2)—current yield and expected capital appreciation—are observable at time zero; they are based on the current yield curve and contain information about differences in expected returns among bonds with different maturities. Term (3) is related to the changes in the yield curve in the future (over the next Δt period) and therefore is not directly observable at time zero. Thus, term (3) cannot be used to identify differences in expected returns *unless* an observable and reliable proxy exists. If such a proxy does not exist, we must determine if the noise introduced by future changes in yield curves is significant or if current yield and expected capital appreciation alone can identify differences in expected returns. To address these issues, we look to the empirical data.

ANALYZING OBSERVABLE (AND UNOBSERVABLE) TERMS OF EXPECTED RETURNS

Finding expected return premiums based on information in current yield curves has been a subject of analysis for at least 40 years. Initial analysis that tried to assess the validity of the expectation hypothesis, which states that the yield curve should evolve over time to allow for only zero or constant term premiums, soundly rejected the hypothesis. As a corollary, the existence of time-varying expected premiums along the yield curve was confirmed.[1]

1. For more information on the expectation hypothesis and the empirical tests, see, for example, Dai (2015).

Taking the logarithm of the expected return equation, the log return in Δt on a bond with duration n can be written as

$$r_{\Delta t}^{(n)} = \Delta t\, y_0^{(n)} - (n-\Delta t)(y_0^{(n-\Delta t)} - y_0^{(n)}) - (n-\Delta t)(y_{\Delta t}^{(n-\Delta t)} - y_0^{(n-\Delta t)})$$

Using the forward rate, $f_0^{(n-\Delta t \to n)}$, the equation above can be expressed as

$$r_{\Delta t}^{(n)} = \Delta t\, f_0^{(n-\Delta t \to n)} - (n-\Delta t)(y_{\Delta t}^{(n-\Delta t)} - y_0^{(n-\Delta t)})$$

The forward rate represents the incremental yield that a bond with n periods to maturity will have relative to a bond with $n - \Delta t$ periods to maturity. In the absence of yield changes in the next Δt, it also represents the bond return over that period of time. As shown in Fama (1976), forward rates can be written as a combination of the expected changes in future yields and the expected term premiums—that is, forward rates have to tell us something about one or the other. We see this by rearranging the equation above and taking expectations on both sides.

Using US Treasury zero coupon yield curve data from 1964 to 2016, we run the following regressions to examine the informational content in forward rates and to determine how much information about expected term premiums and expected changes in future yields is contained in forward rates. The first regression forecasts one-year returns on n-year bonds, while the second regression forecasts (n-1)-year rates one year from now.

As shown in **Exhibit 2**, forward rates contain statistically reliable information about expected term premiums when the forecast horizon is one year. Forward rates tell us very little, however, about changes in yields.[2] In addition, existing forecasting models of yield curve changes have not shown the ability to provide persistent a-posteriori forecasts that are superior to current yields. Current yield curves contain information about future expected changes, and only randomly distributed actual changes with zero mean are observed. In fact, assuming zero expected changes in yield curves seems to perform better than previously proposed models.[3]

2. The results here are consistent with the findings in Fama (1984), Fama and Bliss (1987), and subsequent studies.
3. See, for example, Fama (1976) and Duffee (2002).

Exhibit 2: Informational Content in Forward Rates

Information in the Forward Rates about Term Premiums in the Next Year

$$r_{t+1}^{(n)} - r_{t+1}^{(1)} = \alpha + \beta\left(f_t^{(n-1\to n)} - y_t^{(1)}\right) + \varepsilon_{t+1}^{(n)}$$

n	slope	t-stat	adj. R^2
2	0.79	2.91	0.09
3	0.98	2.97	0.09
4	1.16	3.21	0.11
5	1.34	3.48	0.12

Information in the Forward Rates about Yield Changes in the Next Year

$$y_{t+1}^{(n-1)} - y_t^{(n-1)} = \alpha + \beta\left(f_t^{(n-1\to n)} - y_t^{(1)}\right) + \varepsilon_{t+1}^{(n)}$$

n	slope	t-stat	adj. R^2
2	0.21	0.78	0.01
3	0.01	0.06	0.00
4	−0.05	−0.45	0.00
5	−0.08	−0.88	0.01

Data source: Federal Reserve Board, ***www.federalreserve.gov/pubs/feds/2006/200628/200628abs.html****.*

Given this empirical evidence, we believe a reasonable place to start for government bonds is to use the observable components of expected returns—current yield and expected capital appreciation—when forming portfolios. The optimization problem stated above can be rewritten as

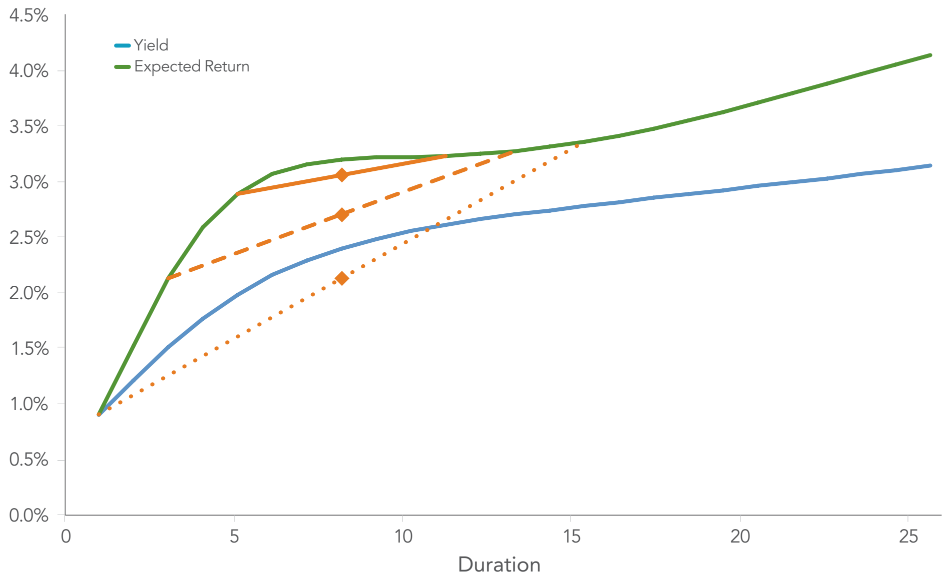
$$\begin{aligned}
&\mathit{Maximize} \sum w_n\, e^{\Delta t\, y_0^{(n)}}\, e^{-(n-\Delta t)(y_0^{(n-\Delta t)} - y_0^{(n)})} \text{ for all } w_n,\ 0 \le n \le \max \mathit{Duration} \\
&\mathit{Subject\ to:} \quad \sum w_n \cdot n = D \\
&\qquad\qquad\quad\ \sum w_n = 1 \\
&\qquad\qquad\quad\ 0 \le w_n \le 1 \text{ for all } w_n
\end{aligned}$$

ILLUSTRATIVE EXAMPLE: US TREASURY ZERO COUPON YIELD CURVE

Under the developed framework, what does a typical shape of the US Treasury zero coupon yield curve imply about the expected returns at different points across the curve and, consequently, which segments to focus on for higher expected returns?

As an illustration, let's look at a hypothetical spot yield curve plotted in blue in **Exhibit 3**. Based on the framework developed in the previous section, we can calculate the expected return using the information in the current yield curve: $E_0\left(R_{\Delta t}^{(n)}\right) = e^{\Delta t\, y_0^{(n)}}\, e^{-(n-\Delta t)(y_0^{(n-\Delta t)} - y_0^{(n)})} - 1$. The resulting expected return curve for $\Delta t = 1Y$ is plotted in green.

Exhibit 3: Expected Return for Hypothetical Yield Curve



For illustrative purposes only. ***Past performance is no guarantee of future results.***

The illustrative example suggests that for the yield curve shape as shown, the expected returns at the short end of the yield curve tend to be lower. Suppose the desired duration is eight years. All else equal, we may want to underweight this maturity segment at the short end in order to increase expected return. We believe the implication is that we should also underweight the long end and focus on the middle part of the curve if we want to keep the duration at eight years.

Exhibit 4 demonstrates that idea with a simple numerical example. While different combinations of points on the curve may have the same average duration, their expected returns can be different. Instead of having exposure to both the short and long ends of the yield curve, our numerical example indicates that focusing on the middle segment of the hypothetical yield curve may lead to higher expected returns.

Exhibit 4: Expected Returns of Different Allocations with Same Duration

Allocation	Duration	Expected Return
50% 1Y + 50% 15Y	8Y	2.13%
50% 3Y + 50% 13Y	8Y	2.69%
50% 5Y + 50% 11Y	8Y	3.05%

Expected returns are based on current yield and expected capital appreciation, i.e., $E_0\left(R_{\Delta t}^{(n)}\right) = e^{\Delta t\, y_0^{(n)}} e^{-(n-\Delta t)(y_0^{(n-\Delta t)} - y_0^{(n)})} - 1$.
Past performance is no guarantee of future results. There is no guarantee that any product or strategy offered by Dimensional will achieve the returns shown. Please see disclosure on last page for important information regarding forward-looking statements.

To empirically test this implication, we use the US Treasury zero coupon yield data from 1982 to 2016[4] and examine the performance of different constant duration portfolios that invest in:

- The section from 1 year to 15 years
- The short end from 1 year to 4 years and the long end from 12 years to 15 years
- The middle segment from 5 years to 11 years

The portfolios rebalance annually in December, and the maturities are equally weighted to achieve duration of eight years at rebalancing time.

As shown in **Exhibit 5**, investing only in the short and long ends of the typical US Treasury zero coupon yield curve has generated an annualized return of 9.00%, which is lower than the 9.21% achieved by investing in 1 year to 15 years. This result is equivalent to saying that the middle segment of the yield curve had the highest return among the three, delivering 9.46% per year over this period. In summary, the empirical results support the developed framework, highlighting how information in yield curves can be used to systematically increase a strategy's expected return, even in the context of having tight duration constraints.

Exhibit 5: Performance of Simulated Constant Duration Portfolios

01/1982–12/2016	Duration	Annualized Compound Return	Annualized Standard Deviation
EW 1Y-15Y	8 Years	9.21%	10.92%
EW 1Y-4Y & 12Y-15Y	8 Years	9.00%	10.78%
EW 5Y-11Y	8 Years	9.46%	11.11%

Data source: Federal Reserve Board, ***www.federalreserve.gov/pubs/feds/2006/200628/200628abs.html****.*

Performance shown is for the time period 1/1/1982–12/30/2016. Past performance is no guarantee of future results. There is no guarantee that any product or strategy offered by Dimensional will achieve the returns shown.

4. This time frame represents all available full-year data of the US Treasury one-year to 20-year zero coupon yields published by the Federal Reserve Board, *www.federalreserve.gov/pubs/feds/2006/200628/200628abs.html*.

CONCLUSION

Research shows that current yield curves contain reliable information about bonds' expected returns. We present a framework underpinning how this information can be used to pursue higher expected returns with duration constraints. By focusing on segments of the yield curve that our research shows offer higher expected returns, we believe investors can increase the return potential of their fixed income investments even when maintaining a desired duration.

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